



DIGITAL WATERMARKING IN THE FRACTIONAL HARTLEY DOMAIN**Vasant Gaikwad**

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ABSTRACT

Digital watermarking aims to balance quality and robustness, crucial for effectiveness, despite their inversely proportional nature. Spatial domain embedding enhances quality but diminishes robustness, whereas frequency domain embedding prioritizes robustness over quality. Utilizing common image transforms like DCT, DFT, and DWT improves robustness, enabling multiple watermark embeddings. The proposed non-blind watermarking algorithm utilizes the DFRHT transform for integration into host and watermark images, evaluated through coefficient addition. Optimization techniques, particularly multi-objective methods, play a vital role in enhancing both robustness and quality via metaheuristic algorithms and fitness function evaluations.

**Subject Classification:** 41A63 ; 42A38 ; 65R10 ; 46F99.**KEY WORDS:** Digital watermarking, Singular Value Decomposition, Fractional Fourier, Fractional Hartley transform, Optimization.**1 INTRODUCTION**

The Hartley transform, proposed by R.V.L. Hartley in 1942, stands as a close counterpart to the Fourier transform. Unlike the Fourier transform, the Hartley transform possesses the advantage of converting real-valued functions into real-valued functions and being its own inverse [12]. An extension of the Hartley transform, the Fractional Hartley transform, parallels the Fractional Fourier transform, an extension of the Fourier transform. Consequently, the Fractional Hartley transform surpasses the Fractional Fourier transform. Emerging in the early decades, the Fractional Fourier transform found its niche in optics and signal analysis, initially introduced by V. Namias in the 1980s from classical quadratic Hamiltonians [8].

This transform finds applications in optical propagation problems [2], time-frequency representations [3], and various scientific endeavors [4, 6– 10, 13]. The paper titled "Optimized SVD-based robust watermarking in the fractional Fourier domain" by A.M. Abdelhakim, M.H. Saad, M. Sayed, and H.I. Saleh explores robust watermarking methods utilizing the Fractional Hartley transform (FRHT) [1]. Continuously derived definitions of the Fractional Fourier transform and the Fractional Hartley transform satisfying boundary conditions and additive properties utilize the general theory of linear fractional transforms [11].

The selection of the FRHT in this paper stems from its fractional power variable, facilitating spatial-frequency representations in phase image encryption [14], nonlinear optical double image encryption [15], and optical image encryption [5, 16, 17].

The evaluation of watermarking methods hinges significantly on two key factors: quality and robustness. These metrics gauge the efficacy of any watermarking approach, favoring techniques that minimize distortion. Within the realm of image watermarking, any form of alteration, such as compression, noise, or filtering, is deemed an attack. Quality and robustness are perpetually at odds; increasing robustness typically entails a decrease in watermarking quality, and vice versa. Striking a balance between robustness and high watermarking quality poses a challenge due to their conflicting nature. While embedding in the spatial domain of the image yields high watermarking quality but low robustness, embedding in the frequency domain enhances robustness at the expense of quality.

Image transforms like DFT (Discrete Fourier Transform), DCT (Discrete Cosine Transform), and DWT (Discrete Wavelet Transform) are commonly employed in watermarking methods to bolster robustness. The method described in this paper offers the capability to embed multiple watermarks by utilizing additional degrees of freedom. The authors introduce a non-blind watermarking algorithm employing the DFRHT transform applied to both the host and watermark images. Embedding is achieved by adding watermark coefficients to image coefficients. The method assesses watermarking performance by studying the effect of watermark capacity on the quality of the watermarked image.

Watermarking optimization is typically employed to enhance both quality and robustness. Multi-objective optimization methods are utilized to address multiple optimization objectives, often implemented in meta-heuristic algorithms using fitness functions to evaluate solutions. The method proposed in this paper achieves optimized quality and robustness by searching for the best wavelet sub-bands and number of coefficients for watermark embedding. Employing singular-value decomposition (SVD) based watermarking, the method scales singular values of the cover image using optimized scaling factors to maximize image quality. The optimization process employs a simple algorithm with a fitness function evaluated using the exponential weighted criterion, a multi-objective optimization scheme.

Utilizing both spatial and frequency domains enhances security and robustness, achieved by applying embedding modifications to singular values of transformed images using SVD. The embedding parameters are evaluated using the ABC algorithm through quality-guaranteed watermarking optimization, providing a practical approach for multimedia applications with quality requirements.

The subsequent sections of the paper are structured as follows: Section II presents the preliminaries, outlining the FRHT and SVD. Section III delineates the proposed methodology, and Section IV concludes the paper.

2 Preliminaries

2.1 Fractional Hartley Transform

The Fractional Hartley Transform (FRHT) serves as a generalization of the Hartley transform, widely employed in various signal processing applications such as phase image encryption [13]. Initially proposed by Soo-Chang Pei, Chien-Cheng Tseng, Min-Hung Yeh, and Ding Jian-Jiun [10], the FRHT utilizes an order parameter to determine the rotation in the time-frequency plane. Consequently, the FRHT transforms a signal $h(t)$ to $g_H^\alpha(v)$, representing an intermediate domain between time and frequency, as described in [10].

$$g_H^\alpha(v) = FRHT_\alpha(h(t)) = \int_{-\infty}^{\infty} K_H^\alpha(t, v)h(t)dt, \tag{2.1}$$

where

$$K_H^\alpha(t, v) = \sqrt{\frac{1 - i \cot \psi}{2\pi}} e^{i\frac{1}{2}(t^2+v^2) \cot \psi} [\cos(tv \csc \psi) + e^{i(\psi-\frac{\pi}{2})} \sin(tv \csc \psi)];$$

and $\psi = \frac{\alpha\pi}{2}$ if $\psi \neq \pi n$; for all $n = 0, 1, 2, \dots$. It's important to note that when α equals 1, the FRHT reduces to the ordinary Hartley transform, resulting in a transformed signal purely in the frequency domain. Conversely, when α equals 0, the transformed signal resides in the time domain. This additional degree of freedom provided by the order parameter α distinguishes the FRHT from the Hartley transform. Furthermore, the inverse of the FRHT is computed using $FRHT(-\alpha)$, which involves applying the transform with an order parameter equal to $-\alpha$, as depicted in the following relation:

$$h(t) = FRHT_{-\alpha}(g_H^\alpha(v)) = \int_{-\infty}^{\infty} g_H^\alpha(v)K_H^{-\alpha}(t, v)dv. \tag{2.2}$$

For two dimensional signals, like images of size MN, the 2d-DFRHT (Discrete Fractional Hartley Transform) can be executed using row-column computation [*] as follows:

$$g_H^{\alpha\beta}(m, n) = \sum_{p=0}^{M-1} \left(\sum_{q=0}^{N-1} h(p, q)K_H^\beta(q, n) \right) K_H^\alpha(p, m), \tag{2.3}$$

where α and β are the two dimensional order parameters.

2.2 Singular Value decomposition

The Singular Value Decomposition (SVD) serves as a pivotal matrix factorization technique with diverse applications such as matrix approximation, pseudo-inverse computation, and linear equation solving. This method facilitates the decomposition of any matrix into three constituent matrices.

$$B = UDV', \tag{2.4}$$

The unitary matrices U and V, denoted as left and right singular vectors respectively, satisfy the conditions $UU^T = I$ and $VV^T = I$. The matrix D is diagonal and contains the singular values of B, obtained by computing the eigenvalues of BB^T . This can be repre

sented as: $D = \begin{pmatrix} d_1 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & d_j & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \dots & 0 \end{pmatrix}$, where j is the rank of the matrix B . Note that

$d_1 > d_2 > \dots > d_j$, i.e. d_1 is the largest singular value.

In image watermarking, the SVD is commonly utilized because of the following characteristics:

The singular values encapsulate fundamental algebraic image properties.

- It can be employed for both square and rectangular images.
- The singular values of an image remain relatively unaffected by general image processing operations such as compression or filtering, thereby enhancing robustness against attacks.
- Achieving high watermarking quality is feasible when embedding is executed by subtly modifying the singular values.

2.3 Artificial Bee Colony (ABC) optimization algorithms

The Artificial Bee Colony (ABC) algorithm is a straightforward population-based optimization approach that emulates the foraging behavior of a bee swarm. In this algorithm, each food source corresponds to a viable solution for the optimization problem, with the quality of the source reflecting the fitness of the associated solution. The foraging process comprises three categories of bees:

- Employed bees: are responsible for gathering food from food sources and exploring the vicinity of each source. They then communicate their findings to the onlooker bees awaiting in the hive.
- Onlookers: select a food source to exploit based on the information provided by the employed bees. Upon discovering a superior food source, they inform the corresponding employed bee to update its position accordingly.
- Scouts: When the update process fails to identify superior sources after several attempts, indicating the algorithm's convergence to a local optimum, the bees explore new food sources randomly. In this scenario, the employed bee transitions into a scout and initiates a new search randomly.

3 THE PROPOSED WATERMARKING METHOD

This section elaborates on the proposed optimized watermarking scheme, which involves inserting a binary watermark into a host image. The embedding process takes place in the FRHT domain, incorporating a fractional power (order parameter) into the watermarking parameters. Additionally, the SVD is utilized to decompose the FRHT coefficients matrix, allowing for embedding modifications based on the singular values. The proposed method enhances security through the utilization of a key necessary for watermark extraction. Embedding of the watermark bits occurs through block-wise operations, where each image block is modified to accommodate a single bit. Figure 1 illustrates a general representation of the proposed watermarking method, incorporating optimization to determine the optimal embedding parameters.

3.1 Embedding process

The subsequent description outlines the watermarking procedure for embedding a watermark bit $w_{r,c}$ located at the r^{th} row and c^{th} column of the watermark image.

- Partition the host image I , sized $R \times C$, into blocks of $m \times n$.
- Perform the FRHT transform on the image block $I_{r,c}$ as follows:

$$I_{r,c}^{FRHT} = FRHT(I_{r,c}, \alpha, \beta), \quad (3.1)$$

where α, β are the FRHT fractional powers, $r = 1, \dots, \frac{R}{8}$, and $c = 1, \dots, \frac{C}{8}$.

- Factorize the transformed image block $I_{r,c}^{FRHT}$ using SVD in the following manner:

$$[U_{r,c} D_{r,c} V_{r,c}] = SVD(I_{r,c}^{FRHT}). \quad (3.2)$$

- Modify the embedding according to $w_{r,c}$ on the first singular value $d_{r,c}^1$ of the $D_{r,c}$ matrix as follows:

$$dw_{r,c}^1 = d_{r,c}^1 + \sigma(-1)^{(1-w_{r,c})}. \quad (3.3)$$

Here, $dw_{r,c}^1$ represents the modified singular value. The scaling factor σ is determined through an optimization algorithm.

- Update the $D_{r,c}$ matrix of the FRHT-transformed image block to $Dw_{r,c}$ by replacing $d_{r,c}^1$ with $dw_{r,c}^1$.

- Recompose the FRHT-transformed image block using the following relation:

$$I_{r,c}^{WFRHT} = U_{r,c} Dw_{r,c} V_{r,c}' \quad (3.4)$$

here $I_{r,c}^{WFRHT}$ represents the FRHT transformed watermarked image block.

- Perform the IFRHT (Inverse FRHT) on the transformed watermarked block as follows:

$$I_{r,c}^W = IFRHT(I_{r,c}^{WFRHT}, \alpha, \beta), \quad (3.5)$$

where $I_{r,c}^W$ is the block at the r^{th} row and the c^{th} column of the watermarked image I^W .

- Generate a key containing the singular value $d_{r,c}^1$ and the fractional power α, β .

The watermarking process concludes when all the watermarked bits are embedded into their respective image blocks using the described embedding steps, where $w_{r,c}$ is embedded into $I_{r,c}$ for $1 < r < R/8$ and $1 < c < C/8$.

3.2 Extraction process

The extraction process of the watermark bit $w_{r,c}$ is outlined below:

- Partition the watermarked host image I^W into $m \times n$ blocks.
- Retrieve the values of the fractional powers α, β and the singular values $d_{r,c}^1$ from the key.
- Execute the FRHT transform on the watermarked image block $I_{r,c}^W$ as follow:

$$I_{r,c}^{WFRHT} = FRHT(I_{r,c}^W, \alpha, \beta) \quad (3.6)$$

- Factorize the transformed block $I_{r,c}^{WFRHT}$ using SVD as follows:

$$[U_{r,c} Dw_{r,c} V_{r,c}'] = SVD(I_{r,c}^{WFRHT}). \quad (3.7)$$

- Get the first singular value $dw_{r,c}^1$ from the singular matrix $Dw_{r,c}$.
- Extract the watermark bit $w_{r,c}$ according the following relation:

$$w_{r,c} = \begin{cases} 1 & \text{if } dw_{r,c}^1 \geq d_{r,c}^1 \\ 0 & \text{otherwise} \end{cases} \quad (3.8)$$

4 CONCLUSION

This paper introduces an optimized watermarking method employing a novel embedding algorithm that utilizes the 2D-FRHT and SVD. Meta-heuristic optimization is employed to identify the optimal solution for the inherent trade-off between watermark imperceptibility and robustness. The FRHT fractional strength and the embedding power parameter are determined through quality-guaranteed watermarking optimization using the ABC algorithm. The approach ensures maximal

robustness while maintaining watermarking quality above a predefined threshold. Results demonstrate that the proposed watermarking technique achieves superior quality and robustness compared to recent methods, particularly against various types of attacks.

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