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MODIFIED RATIO TYPE ESTIMATOR OF TWO POPULATION MEANS IN STRATIFIED SAMPLING

$$\bar{y}_{st\_a} = \sum_{h=1}^{L} \frac{N_h}{n_h} \bar{w}_{yh}$$

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Abstract:-A modified ratio estimator is proposed for ratio of two population means using auxiliary information in stratified random sampling. Bias and mean square error expression are obtained and the proposed estimator is compared theoretically and empirically with classical estimator.

Keywords: Ratio estimator, auxiliary information, stratified sampling, bias and mean squared error.

INTRODUCTION :

Stratified sampling comes under the category of restricted sampling. Stratified sampling is generally used when population is heterogeneous. In this sampling method, the population is subdivided into subpopulations (homogeneous groups under certain criteria) called strata. The main objective of stratification is to give a better cross-section of the population, so as to gain a higher degree of relative precision. Many researchers have studied the estimation of the ratio of two population means in simple random sampling(Singh,1965; Rao& Pareira,1968; Shah & Shah,1978;Ray & Singh,1985; Upadhyaya& Singh,1985; Upadhyaya, et al.1985; Singh & Rani,2005,2006; Sindhu, et al.,(2009). Other sampling designs have not attracted much attention; in many situations, it has been observed that stratified random sampling provides efficient estimators compared to those of simple random sampling.

Consider a finite population, $P = P1, P2, \ldots\ldots..PN$ of size N. This population P is divided into L strata each of size N_h and sample of size n_h is drawn from each stratum such that $n = \sum_{i=1}^{L} n_h$

(h=1,2,……,L).If y_0 and y_1 are the variates, x is an auxiliary variate, and y_{0hi}, y_{1hi} ,and x_{hi}(h=1,2,…..,L; i=1,2,…..,N_h) are the observations taken from the i^{ih} unit of the h^{th} stratum on study variates y_0 y_1 and auxiliary variate x respectively, then the following are defined:

h^{th} stratum mean for study variate y_0:

$$\bar{y}_{0h} = \frac{1}{N}\sum_{i=1}^{Nh} y_{0hi}$$

h^{th} stratum mean for study variate y_1:

$$\bar{Y}_{1h} = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{1h1}$$

h^{th} stratum mean for auxiliary variate x:

$$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{1h1}$$

Population mean of study variate y_0:

$$\bar{Y}_0 = \frac{1}{N}\sum_{h=1}^{L} N_h \bar{Y}_{0h} = \sum_{h=1}^{L} W_h \bar{Y}_{0h}$$

Population mean of study variate y_1:

$$\bar{Y}_1 = \frac{1}{N}\sum_{h=1}^{L} N_h \bar{Y}_{1h} = \sum_{h=1}^{L} W_h \bar{Y}_{1h}$$

Population mean of auxiliary variate x:

$$\bar{X} = \frac{1}{N}\sum_{h=1}^{L} W_h \bar{X}_h$$

Sample mean of study variate y_0 for h^{th} stratum:

$$\bar{y}_{0h} = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{0hi}$$

Sample mean of study variate y_1 for h^{th} stratum:

$$\bar{y}_{1h} = \frac{1}{n_h}\sum_{i=1}^{n_h} \bar{y}_{1h1}$$

Sample mean of auxiliary variate x for h^{th} stratum:

$$\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} \bar{x}_{hi}$$

Stratum weight of h^{th} stratum:

$$W_h = \frac{N_h}{N}$$

Ratio of two population means:

$$R = \frac{\bar{Y}_0}{\bar{Y}_1}$$

The usual unbiased estimators of population means $\bar{Y}_0, \bar{Y}_1 and, \bar{X}$ are

$$\bar{y}_{0st} = \sum_{h=1}^{L} W_h \bar{y}_{0h}$$

$$\bar{y}_{1st} = \sum_{h=1}^{L} W_h \bar{y}_{1h}$$

$$\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$$

And the estimator of the ratio of two populations means R in stratified random sampling is:

$$\hat{R}_{st} = \left(\frac{\bar{y}_{0st}}{\bar{y}_{1st}}\right)$$

Proposed ratio estimator

When the population mean of the auxiliary variable $\bar{X}$ is known, Singh (1965) suggested an estimator for R in simple random sampling as,

$$G_1 = \left(\frac{\bar{y}_0}{\bar{y}_1}\right)\left(\frac{\bar{X}}{\bar{x}}\right) = R\left(\frac{\bar{X}}{\bar{x}}\right)$$

In stratified random sampling G^{1st} is defined as

$$G_{1st} = \left(\frac{\bar{y}_{0st}}{\bar{y}_{1st}}\right)\left(\frac{\bar{X}}{\bar{x}_{st}}\right)$$

On the above lines, we propose a modified estimator using the median of the auxiliary variable in combination with the auxiliary mean

$$G_{ARst} = \left(\frac{\bar{y}_{0st}}{\bar{y}_{1st}}\right)\left(\frac{\bar{X}+Md}{\bar{x}_{st}}\right)$$

In order to derive the bias and mean square error expressions of G_{1st} it is assumed that,

$\bar{y}_{0h} = \bar{Y}_{0h}(1+e_{0h})$, $\bar{y}_{1h} = \bar{Y}_{1h}(1+e_{1h})$ and $\bar{x}_h = \bar{X}(1+e_{2h})$ such that $E(e_{0h}) = E(e_{1h}) = E(e_{2h}) = 0$

To the first degree of approximation the bias and mean square error are

$$B(G_{1ST}) = R\sum_{h=1}^{L} W_h^2 \gamma_h \left[\frac{S_{1h}^2}{\bar{Y}_1^2} + \frac{S_{xh}^2}{\bar{X}^2} - \frac{S_{01h}}{\bar{Y}_0\bar{Y}_1} - \frac{S_{0xh}}{\bar{Y}_0\bar{X}} + \frac{S_{1xh}}{\bar{Y}_1\bar{X}}\right] \quad (2.1)$$

$$B(G_{ARst}) = R\sum W_h^2 \gamma_h \left[\frac{S_{1h}^2}{\bar{Y}_1^2} + \frac{S_{xh}^2}{(\bar{X}+Md)^2} - \frac{S_{01h}}{\bar{Y}_0\bar{Y}_1} - \frac{S_{0xh}}{\bar{Y}_0(\bar{X}+Md)} + \frac{S_{1xh}}{\bar{Y}_1(\bar{X}+Md)}\right] \quad (2.2)$$

$$MSE(G_{1st}) = R^2\sum_{h=1}^{L} W_h^2 \gamma_h \left\{\frac{S_{0h}^2}{\bar{Y}_0^2} + \frac{S_{ih}^2}{\bar{Y}_1^2} + \frac{S_{xh}^2}{\bar{X}^2} - 2\left(\frac{S_{01h}}{\bar{Y}_0\bar{Y}_1} + \frac{S_{0xh}}{\bar{Y}_0\bar{X}} - \frac{S_{1xh}}{\bar{Y}_1\bar{X}}\right)\right\} \quad (2.3)$$

$$MSE(G_{ARst}) = R^2\sum_{h=1}^{L} W_h^2 \gamma_h \left\{\frac{S_{oh}^2}{\bar{Y}_0^2} + \frac{S_{1h}^2}{\bar{Y}_1^2} + \frac{S_{xh}^2}{(\bar{X}+Md)^2} - 2\left(\frac{S_{01h}}{\bar{Y}_0\bar{Y}_1} + \frac{S_{0xh}}{\bar{Y}_0(\bar{X}+Md)} - \frac{S_{1xh}}{\bar{Y}_1(\bar{X}+Md)}\right)\right\} \quad (2.4)$$

Where

$$S_{0h}^{2} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{0hi} - \bar{y}_{1h})^2$$
$$S_{1h}^{2} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{1hi} - \bar{y}_{1h})^2$$
$$S_{xh}^{2} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(x_{hi} - \bar{x}_h)^2$$ and
$$S_{01h} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{0hi} - \bar{y}_{0h})(y_{1hi} - \bar{y}_{1h})$$
$$S_{0xh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{0hi} - \bar{y}_{0h})(x_{hi} - \bar{x}_h)$$
$$S_{1xh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{1hi} - \bar{y}_{1h})(x_{hi} - \bar{x}_h)$$

Efficiency Comparison of Estimators

A comparison of equations (2.3) and (2.4) shows that the suggested estimator G_{ARst} will be more efficient than G_{1st} if

$$MSE(G_{ARst}) - MSE(G_{1st}) < 0$$

Empirical Study

For numerical illustration we have taken the data from Murthy (1967) in which X represents the output y_0 denotes number of workers and y_2 denotes the fixed capital.

	n_1=2	n_2=2	N_1=5	N_2=5
	$\bar{Y}_{01} = 91.4$	$\bar{Y}_{02} = 109.2$	$\bar{Y}_{11} = 577.8$	$\bar{Y}_{12} = 609.6$
N=10	$\bar{X}_1 = 3757$	$\bar{X}_2 = 4049.6$	$S_{y01} = 4.0373$	$S_{y02} = 6.1400$
n=4	$S_{y11} = 38.78402$	$S_{y12} = 43.0848$	$S_{x1} = 40.5401$	$S_{X2} = 126.7292$
	$S_{011} = 84.6$	$S_{012} = 245.1$	$S_{0x1} = 139.5$	$S_{0x2} = 761.35$
	$S_{1x1} = 860.5$	$S_{1x2} = 4597.55$	Md(x)=3853.5	

Therefore the Mean Square Error and percent relative efficiency of the proposed estimator comes out to be 0.019 and 310.09 respectively.

CONCLUSION:

From the above results , we conclude that the proposed estimator suggested by the authors performed well in all situations and when information regarding population mean is available , the estimator is recommended for use in practice.

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