

Vol II Issue XI Dec 2012

ISSN No : 2230-7850

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Monthly Multidisciplinary  
Research Journal

# *Indian Streams Research Journal*

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**RNI MAHMUL/2011/38595**

**ISSN No.2230-7850**

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## SUPERCONDUCTIVITY FROM WEAK TO STRONG COUPLING IN THE ATTRACTIVE HUBBARD MODEL: AN OVERVIEW

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### Abstract:

*In this review paper we will complement the analysis of the phase diagram to finite temperature. But, still using exact diagonalization (ED) to solve the impurity model associated with the DMFT of the Hubbard model. We also compare the normal-state solutions with the superconducting solutions which are stable at low temperatures. The use of ED allows us to reach arbitrarily small temperatures, which are hardly accessible by means of QMC. Quite naturally, the extension of ED to finite temperature requires a more severe truncation of the Hilbert space.*

### INDEX-TERMS :

DMFT, Hubbard model.& Bose-Einstein (BE) superconductivity

### INTRODUCTION

The attractive Hubbard model represents an invaluable tool to understand properties of pairing and superconductivity in systems with attractive interactions. The simplifications introduced in this model allow a comprehensive study of the evolution from the weak-coupling regime, where superconductivity is due to BCS pairing in a Fermi liquid phase, and a strong-coupling regime, in which the system is better described in terms of bosonic pairs, whose condensation gives rise to superconductivity (Bose-Einstein (BE) superconductivity) [1]. It has been convincingly shown that such an evolution is a smooth crossover and the highest critical temperature is achieved in the intermediate regime where none of the limiting approaches is rigorously valid [1, 2]. A realization of such a crossover scenario has been recently obtained through the development of experiments on the condensation of ultracold trapped fermionic atoms [3]. In these systems, the strength of the attraction can be tuned by means of a tunable Fano-Feshbach resonance, and the whole crossover can be described [4].

In the context of high-temperature superconductivity, the intermediate-strong coupling regime in which incoherent pairs are formed well above the critical temperature has been invoked as an interpretation of the pseudogap phase [5]. Moreover, since the early days of the discovery of these materials, the evolution of both the normal- and the superconducting-phase properties with the doping level made some authors [6] to recognize the fingerprints of a crossover between a relatively standard BCS-like superconductivity in the overdoped materials and a strong-coupling superconductivity associated with BE condensation in the underdoped materials. Indeed, at optimal doping, the zero-temperature coherence length is estimated to be about 10-20 Å [7, 8], i.e., much smaller than for conventional superconductors but still large enough to exclude the formation of local pairs [9, 10]. It is understood that the attractive Hubbard model should not be taken as a microscopic model for the cuprates, since a realistic description of the copper-oxygen planes of

these materials unavoidably requires a proper treatment of strong Coulomb repulsion. This simplified model represents instead an ideal framework where the evolution from weak to strong coupling can be studied by simply tuning the strength of the attraction. The main aim of the present work is to determine whether, and to what extent, at least some aspects of the phenomenology of the cuprates can be interpreted simply in terms of a crossover from weak to strong coupling.

## DISCUSSION

The main simplifications introduced by the attractive Hubbard model can be summarized as follows:

- (i) neglect of repulsion. Even if some attraction has to develop at low energy, the large short-range Coulomb repulsion implies that the interaction must become repulsive at high energy in real systems. In some sense, an attractive Hubbard model picture can at the most be applied to the low-energy quasi particles.
- (ii) (ii) The model naturally presents s-wave superconductivity, as opposed to the d-wave symmetry observed in the cuprates.
- (iii) (iii) Neglect of retardation effects. The Hubbard model describes instantaneous interactions, while every physical pairing is expected to present a typical energy scale. For this purpose a model will be considered.

The limiting regimes on equal footing overcoming the drawbacks of perturbative expansions. quantum Monte Carlo (QMC) simulations represent a valuable tool in this regard, and they have been applied to the two- [2], [11]-[14] and three-dimensional [15] attractive Hubbard model. Even if the sign problem does not affect these simulations, finite-size effects and memory requirements still partially limit the potentiality of this approach.

A different non-perturbative approach is the dynamical mean-field theory (DMFT), which neglects the spatial correlations beyond the mean-field level in order to fully retain the local quantum dynamics and becomes exact in the limit of infinite dimensions [16]. Due to the local nature of the interaction in the attractive Hubbard model, we expect that the physics of local pairing is well described in DMFT. Moreover, this approach is not biased towards metallic or insulating states, and it is therefore particularly useful to analyse the BE-BCS crossover. On the other hand, the simplifications introduced by the DMFT are rigorously valid only in the infinite-dimensionality limit, and even if the DMFT has obtained many successes for three-dimensional systems. In particular, the role of dimensionality in determining the pseudogap properties of the attractive Hubbard model has been discussed in [7].

The study of the attractive Hubbard model can greatly benefit from mapping it onto a repulsive model in a magnetic field. The mapping is realized in a bipartite lattice [8] by a 'staggered' particle-hole

transformation on the down spins  $c_{i\downarrow} \rightarrow (-1)^i c_{i\downarrow}$ . The attractive model with a finite density  $n$  transforms into a half-filled repulsive model with a finite magnetization  $m = n - 1$ . The chemical potential is transformed, accordingly, into a magnetic field  $h = \mu$ . In the  $n = 1$  case (half-filling), the two models are therefore completely equivalent. We notice that the above mapping not only holds for the normal phases, but extends to the broken symmetry solutions. The three components of the antiferromagnetic order parameter of the repulsive Hubbard model are in fact mapped onto a staggered charge-density-wave parameter ( $z$  component of the spin) and an s-wave superconducting order parameter ( $x$ - $y$  components). The above mapping is extremely useful, since it allows us to exploit all the known results for the repulsive model and for the Mott-Hubbard transition to improve our understanding of the attractive model.

. In particular, a phase transition has been found both at finite [19] and at zero temperature [20] between a metallic solution and a pairing phase of pairs. The insulating-pairs phase is nothing but a realization of a superconductor without phase coherence, i.e., a collection of independent pairs. As it has been discussed in [19, 20], this phase is the 'negative- $U$ ' counterpart of the paramagnetic Mott insulator found for the repulsive Hubbard model. We notice that the insulating character of the pairing phase is a limitation of the DMFT approach in which the residual kinetic energy of the preformed pairs is not described. The pairing transition has been first identified in [19] by means of a finite temperature QMC solution of the DMFT. The  $T = 0$  study of [20] has clarified that the pairing transition is always of first order except for the half-filled case, and that it takes place with a finite value of the quasi-particle weight  $Z = \{1 - [\Sigma(\omega)/\omega]\} - 1$ , associated with a finite spectral weight at the Fermi level. As to the onset of superconductivity, a DMFT calculation of the critical temperature  $T_c$  has been performed in [19] for the case of  $n = 0.5$ . The  $T_c$  curve, extracted from the divergence of the pair-correlation function in the normal phase, displays a clear maximum at intermediate coupling and reproduces correctly both the BCS and the

BE predictions in the asymptotic limits, remaining finite for all  $U \rightarrow 0$ .

## METHOD

In this research project, we will look at the destruction of the superconducting state in the attractive Hubbard model through the systematic inclusion of  $U=0$  sites. A consistent picture will emerge from an analysis of the pairing structure factor and the super fluid fraction, namely that pairing survives out to a dilution of about  $3/10$  of the attractive sites, and that this value is not very dependent on the filling, although it may be somewhat increased away from half filling, this result is in marked contrast with data obtained within the coherent potential approximation, which will show a linear dependence of  $f_{\text{crit}}$  on the density. As the fraction  $f$  of  $U=0$  sites will increase the high temperature peak in  $C(T)$  which signals local pair formation, will remain relatively unchanged. The low  $T$  peak which signals the establishment of pairing order gets pushed down. Although the inclusion of noninteracting sites might remind one of percolation, the transition here is not percolative. The critical value of  $f$  is neither that of classical site percolation nor of quantum percolation. Finally, we will show that directly at half filling, pairing correlations are enhanced by the inclusion of vacancy sites. We will attribute this effect to the breaking of degeneracy of the CDW and superconducting order in favour of pairing. The obtained results will be compared with the available literature so that the comment will be made for their suitability.

## OBJECTIVE

Hubbard model exists, both on finite lattices and within dynamical mean field theory where the momentum dependence of the self energy is neglected. Several interesting effects, including the possibility of the enhancement of  $N_{\text{eff}}$  by site disorder, a Mott transition away from half filling, and a change in the temperature dependence of resistivity from insulating to metallic behavior when interactions are turned on, have been predicted.

## SUMMARY

The DMFT extends the concept of classical mean-field theories to quantum problems, by describing a lattice model in terms of an effective dynamical local theory. The latter can be represented through an impurity model subject to a self-consistency condition, which contains all the information about the original lattice structure through the non-interacting density of states (DOS) [16]. Starting with the Hubbard model (1), we obtain an attractive Anderson impurity model.

## ACKNOWLEDGEMENT

I am very much grateful to University Grants Commission ERO, Kolkata for the financial assistance (Minor Research Project). I am also grateful to Physics Dept. of C.M.Sc. College, Darbhanga-846004 for providing the all necessary facilities for my project.

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