INDIAN STREAMS RESEARCH JOURNAL
ISSN NO : 2230-7850
IMPACT FACTOR : 5.1651 (UIF)
VOLUME - 12 | ISSUE - 8 | SEPTEMBER - 2022

SETS, RELATIONS AND FUNCTIONS

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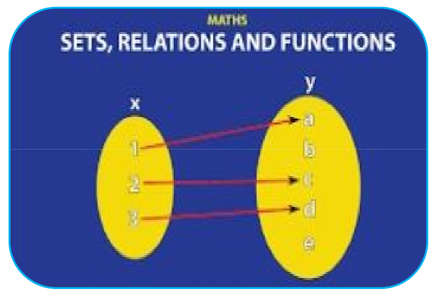

INTRODUCTION:

In mathematics, "sets, relations and functions" is one of the most important topics of set theory. Sets, relations and functions are three different words having different meaning mathematically but equally important for the preparation of JEE mains. In this article, let us study the topic in detail for better understanding.

KEYWORDS : sets, relations and functions, set theory.

SETS IN MATHS:

A set is a collection of well defined objects. The objects of a set are taken as distinct only on the ground of simplicity.

A set of sets is frequently called a family or collection of sets. For example, suppose we have a family of sets consisting A_1, A_2, A_3,….. up to A_n, that is the family {A_1, A_2, A_3,….., A_n } and could be denoted as

S = {A_i | i belongs to N and $1 \le i \le n$}

Notation: A set is denoted by a capital letter and represented by listing all its elements between curly brackets such as { }.

Types of Sets:

In sets theory, there are many types of sets. Some of them are discussed below.

Singleton set:

A set contains only one element. For example, A = {3} and B = {pencil}. Here A and B are containing only one element so both are singleton sets.

Empty Set/Null Set:

An empty set is a set with no element. It is denoted by A = { } or A = ϕ.

Proper set

If A and B are two sets, then A is a proper subset of B if $A \subseteq B$ but $A \neq B$.
For example, if B = {2, 3, 5} then A = {2, 5} is a proper subset of B.

Power Set

The collection of all subsets of a set is the power set of that set. If A is the set then P(A) is denoted as its power set.

The number of elements contained by any power set can be calculated by $n[P(A)] = 2^n$ where n is the number of elements in set A.

For example, If A = {1, 2} then, P(A) = {Ø, {1}, {2}, {1, 2}}
Number of elements in P(A) = 2^2 = 4

Finite Set

A set contains finite number of elements. For example: A = { 2, 4, 6, 8, 10} and B = { a, v, t}. There are 5 objects in set A and 3 elements contained by set B.

Infinite set

If the number of elements in a set is infinite, the set is called an infinite set. For example, N = set of whole numbers = { 0, 1, 2, 3, 4, 5, ……}

Universal Set

- Any set which is a superset of all the sets under consideration and usually it is denoted as S or U.
- For example, Let P = {3, 4, 7} and Q = {1, 2, 3} then we take S = {1, 2, 3, 4, 7} as universe set.

Equal Sets

Two sets P and Q are equal if both are a subset of each other.
Mathematically: If $P \subseteq Q$ and $Q \subseteq P$ then P = Q.
For example, P = {3, 6, 8} and Q = {6, 3, 8}
Here P and Q have exactly the same elements. Satisfy the condition $P \subseteq Q$ and $Q \subseteq P$.
Thus
P = Q.

Operations on Sets

In sets theory, there are basically three operations applicable on two sets are

- Union of two sets
- Intersection of two sets
- Difference of two sets

Relations in Maths

- Relation is helpful to find the relationship between **input** and **output** of a function.
- A relation R, from a non-empty set P to another non-empty set Q, is a subset of P X Q.
- For example, Let P = {a, b, c} and Q = {3, 4} and
- Let R = {(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)}
- Here R is a subset of A x B. Therefore R is a relation from P to Q.

Types of Relations

Empty Relation

An empty relation (or void relation) is one in which there is no relation between any elements of a set. For example, if set A = {1, 2, 3} then, one of the void relations can be R = {x, y} where, |x – y| = 8.
For empty relation,
$R = \varphi \subset A \times A$

Universal Relation

A universal (or full relation) is a type of relation in which every element of a set is related to each other. Consider set A = {a, b, c}. Now one of the universal relations will be R = {x, y} where, |x – y| ≥ 0. For universal relation,
R = A × A

Identity Relation

In an identity relation, every element of a set is related to itself only. For example, in a set A = {a, b, c}, the identity relation will be I = {a, a}, {b, b}, {c, c}. For identity relation,
I = {(a, a), a ∈ A}

Inverse Relation

Inverse relation is seen when a set has elements which are inverse pairs of another set. For example if set A = {(a, b), (c, d)}, then inverse relation will be R^{-1} = {(b, a), (d, c)}. So, for an inverse relation,
R^{-1} = {(b, a): (a, b) ∈ R}

Reflexive Relation

In a reflexive relation, every element maps to itself. For example, consider a set A = {1, 2,}. Now an example of reflexive relation will be R = {(1, 1), (2, 2), (1, 2), (2, 1)}. The reflexive relation is given by-
(a, a) ∈ R

Symmetric Relation

In a symmetric relation, if a=b is true then b=a is also true. In other words, a relation R is symmetric only if (b, a) ∈ R is true when (a,b) ∈ R. An example of symmetric relation will be R = {(1, 2), (2, 1)} for a set A = {1, 2}. So, for a symmetric relation,
aRb ⇒ bRa, ∀ a, b ∈ A

Transitive Relation

For transitive relation, if (x, y) ∈ R, (y, z) ∈ R, then (x, z) ∈ R. For a transitive relation,
aRb and bRc ⇒ aRc ∀ a, b, c ∈ A

Equivalence Relation

If a relation is reflexive, symmetric and transitive at the same time it is known as an equivalence relation.

Domain, Co-domain and Range of a Relation

Let A and B are two sets. In domain, co-domain and range of a relation, if R be a relation from A to B then

- Domain of relation R (Dom(R)) is the set of all those elements a ∈ A such that (a, b) ∈ R for some b ∈ B.
- If R be a relation from A to B, then B is the co-domain of R.
- Range of relation R is the set of all those elements b ∈ B such that (a, b) ∈ R for some a ∈ A.

In short: Domain = Dom(R) = {a : (a, b) ∈ R} and Range (R) = {b : (a, b) ∈ R}

Note: Range is always a subset of co-domain.

Functions in Maths

A function is simply used to represent the dependence of one quantity on the other and easily defined with the help of the concept of mapping. In simple words, a function is a relation which derives one **output** for each **input**.

A function from set P to set Q is a rule that assigns to each element of set P, one and only one element of set Q.

Mathematically: If f: A -> B where y = f(x), x ∈ P and y ∈ Q. Here y is the image of x under f.

Domain, Co-domain and Range of a Function

A function f from a set P to a set Q, represented as f: P-> Q, is a mapping of elements of P (domain) to elements of Y(co-domain) in such a way that each element of P is assigned to some chosen element of Q. That is every element of *P must be assigned to some element of Q and only one element of Q*.

Domain:

For a function, y = f(x), the set of all the values of x is called the domain of the function. It refers to the set of possible input values.

Range:

Range of y = f(x) is a collection of all outputs f(x) corresponding to each real number in the domain. Range is the set of all the values of y. It refers to the set of possible output values.

For example, consider the following relation.
{(2, 3), (4, 5), (6, 7)}
Here Domain = {2, 4, 6}
Range = { 3, 5, 7}

Relations and Functions Important JEE Main Questions

Solved Examples

Example 1: Find the domain of the below function

Solution:

Here |x – 2| = x – 2 if x ≥ 2 and |x – 2| = -(x – 2) or 2 – x if x < 2
|x – 2| – (x – 2) = 0 if x ≥ 2 and |x – 2| – (x – 2) = 4 – 2x if x < 2
So, the above function is defined for (-infinity, 2)
Domain of the given function is (-infinity, 2).

Example 2: Find domain, co-domain and range of a Relation pRq = p divides q.
If P = {3, 5, 7} and Q = {6, 12, 5, 9}.

Solution:

P = {3, 5, 7} and Q = {6, 12, 5, 9}

Since we have relation between elements of P and Q is "p divides q", then
R = {(3, 6), (3, 12), (3, 9), (5, 5)}

Therefore,
Dom(R) = {3, 5}
Co-domain of R = Q = {6, 12, 5, 9}
Range = {5, 6, 9, 12}

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