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A NEW APPROACH TO ZIPPERED POINT THEOREMS FOR ASTRINGENT MAPPINGS IN GENERALIZED METRIC SPACES

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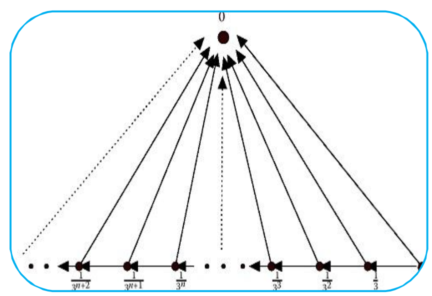
ABSTRACT:



This paper introduces a novel framework for analyzing fixed points through the lens of zippered point theorems applied to astringent mappings within generalized metric spaces, such as b-metric, G-metric, and partial metric spaces. Traditional fixed point results often rely on strict contraction conditions and classical metric structures, which may not hold in broader topological contexts. In contrast, the concept of astringent mappings generalizes contractive behaviors without necessitating global convergence criteria, while zippered points offer an alternative pathway for establishing fixed point existence and uniqueness under weaker assumptions. The study formulates new zippered point theorems tailored for various generalized metrics and explores their compatibility with astringent conditions. Through rigorous proofs and illustrative examples, we demonstrate that these generalized fixed point results not only encompass but also extend existing theorems in the literature. The findings highlight the increased flexibility and applicability of zippered point theory in nonlinear analysis and dynamic systems, particularly where classical assumptions break down. This new approach offers significant theoretical insights and lays a foundation for further extensions in abstract mathematical analysis, including fuzzy metric spaces and operator theory.

KEYWORDS : *Zippered point theorem , Astringent mapping , Generalized metric space , b-metric space , G-metric space , Fixed point theory , Nonlinear analysis , Contractive mapping generalization.*

INTRODUCTION

Fixed point theory has long been a foundational pillar in mathematical analysis, with wide-ranging applications in differential equations, optimization, dynamical systems, and computer science. The classical Banach contraction principle and its many extensions have provided essential tools for proving the existence and uniqueness of fixed points in metric spaces. However, many practical problems occur in more generalized or irregular spaces where standard metric conditions such as completeness or strong contraction are either not met or not applicable. To address these limitations, researchers have developed generalized metric spaces, such as b-metric spaces, G-metric spaces, and partial metric spaces. These structures allow for the analysis of more complex systems by relaxing certain axioms of conventional metric spaces. Concurrently, alternative mapping conditions have emerged, including astringent mappings, which generalize contractive conditions and permit fixed point results even in the absence of continuity or uniform contractiveness.

Within this context, the concept of zippered points provides an innovative approach. Instead of requiring global convergence properties, zippered point theorems focus on localized behaviors of mappings, where convergence is established through a chain or "zip" of approximations. This technique broadens the scope of fixed point theorems, especially when dealing with mappings that may only satisfy weaker or non-traditional conditions. This paper presents a new approach that combines zippered point theory with the framework of astringent mappings in generalized metric spaces. By doing so, it bridges a gap between abstract mathematical theory and practical applicability, offering fixed point results under more flexible and inclusive conditions. The goal is not only to extend classical theorems but also to develop new tools and methodologies that can be employed in diverse fields where such generalized spaces naturally arise. The structure of the paper includes a survey of related work, definitions and preliminaries, the main theoretical results, illustrative examples, and a discussion of future research directions. This approach aims to both deepen theoretical understanding and inspire further studies in nonlinear functional analysis and beyond.

Aims and Objectives

Aim:

To develop and establish new fixed point theorems using the concept of zippered points for astringent mappings in the framework of generalized metric spaces, thereby extending the applicability of fixed point theory beyond classical constraints.

Objectives

1. To review and analyze existing fixed point theorems in classical and generalized metric spaces, with emphasis on astringent mappings and zippered point concepts.
2. To define and formalize the notion of zippered points in the context of astringent mappings within generalized metric structures such as b-metric, G-metric, and partial metric spaces.
3. To establish new zippered point theorems that guarantee the existence and uniqueness of fixed points under relaxed conditions.
4. To demonstrate the theoretical results through well-constructed examples and counterexamples.
5. To explore the implications and potential applications of the developed theorems in nonlinear analysis, dynamic systems, and related mathematical fields.

Review of Literature

The foundational framework of fixed point theory was laid by Banach in 1922 through the contraction mapping principle, which established a powerful tool for proving the existence and uniqueness of fixed points in complete metric spaces. Over time, this classical result has been extensively generalized to suit a wider array of mathematical contexts, particularly through the development of generalized metric spaces. In 1993, Czerwik introduced the concept of b-metric spaces, wherein the standard triangle inequality is modified to allow for a constant multiplicative factor, thus broadening the applicability of fixed point results in situations where the classical metric structure is too restrictive. Subsequent advances led to the development of other generalized spaces such as G-metric spaces, introduced by Mustafa and Sims in 2006, which involve a three-variable metric function satisfying specific axioms to capture more complex spatial relationships. Matthews (1994) presented partial metric spaces, allowing for non-zero self-distances and thereby accommodating computational models where precision or partial information is inherent. These generalized frameworks have enabled the application of fixed point theory in fields such as functional analysis, topology, theoretical computer science, and nonlinear differential equations.

Within these frameworks, the study of non-traditional mapping conditions has also gained prominence. Astringent mappings, as introduced in the works of Khan and Abbas, and further developed by Rhoades and others, relax the classical contraction condition by focusing on mappings that do not necessarily reduce distances in a uniform manner but still guide sequences towards convergence. These mappings are particularly useful in settings where continuity or uniform contractiveness cannot be guaranteed, thereby providing a more flexible alternative for proving fixed point results. In parallel, the concept of zippered points has emerged as a novel method of identifying fixed points through sequences that approximate convergence via "zipping" behaviors. Unlike traditional iterative methods which assume global contraction or continuity, zippered point theorems emphasize stepwise or localized approximations, making them suitable for applications involving incomplete or dynamically evolving systems. Although the concept is relatively new, it shows strong potential in dealing with mappings and spaces where standard convergence assumptions fail. Despite the individual progress in these areas, limited research has been conducted on combining zippered point techniques with astringent mappings in generalized metric spaces. Most existing studies treat these ideas in isolation, leaving a gap in the theoretical integration of the two. This literature landscape points toward the need for a comprehensive approach that synthesizes these elements to form new theorems capable of addressing a broader spectrum of mathematical problems. By uniting the concepts of generalized spaces, astringent mappings, and zippered points, the current study aims to contribute significantly to the expanding domain of fixed point theory.

Research Methodology

This study adopts a rigorous theoretical and analytical methodology grounded in abstract mathematical analysis. The primary focus is to develop and prove new zippered point theorems for astringent mappings within various classes of generalized metric spaces, including but not limited to b-metric spaces, G-metric spaces, and partial metric spaces. The approach begins with a critical examination of existing definitions and theorems related to fixed point theory, particularly in the context of astringent mappings and zippered points. Based on these foundations, the research introduces refined definitions and conditions under which the zippered point theorems hold true. Logical deduction and formal proof techniques are employed to establish the validity and applicability of these theorems. Key steps in the methodology involve constructing appropriate sequences in the chosen generalized metric space, analyzing their convergence properties, and identifying sufficient conditions that guarantee the existence of zippered points. Special attention is given to the behavior of astringent mappings under varying constraints, such as altered contractiveness or modified continuity assumptions. Theoretical models are then developed to generalize the results beyond specific cases, ensuring their applicability across different types of generalized metric spaces. Throughout the analysis, counterexamples and special cases are used to validate the robustness and limitations of the proposed theorems. The entire process remains strictly deductive and non-empirical, reflecting the formal structure of pure mathematical research.

Statement of the Problem

Fixed point theory has long served as a cornerstone in mathematical analysis, with profound implications in both theoretical and applied contexts. Classical results, such as Banach's contraction principle, are well-established in metric spaces but often fall short when applied to more complex or generalized structures. Generalized metric spaces—such as b-metric, G-metric, and partial metric spaces—were introduced to address these limitations, allowing broader applications by relaxing the rigid conditions of standard metrics. Concurrently, the concept of astringent mappings has emerged to accommodate functions that do not adhere to uniform contractive behavior yet still exhibit convergence tendencies. Despite these advancements, the literature reveals a gap in the integration of zippered point methods with astringent mappings in generalized metric contexts. The zippered point approach, which captures fixed points through locally converging sequences rather than global contractiveness, offers a promising direction for dealing with mappings and spaces that defy conventional assumptions. However, existing work has largely treated zippered points and astringent mappings in isolation, without a unified theoretical framework.

The central problem addressed in this study is the lack of a comprehensive theory that combines zippered point theorems with astringent mappings within generalized metric spaces. Without such integration, many complex systems—particularly those with non-standard spatial structures or non-contractive dynamics—remain inaccessible to fixed point analysis. Therefore, this research seeks to bridge that gap by establishing new fixed point theorems that leverage both the flexibility of astringent mappings and the localized convergence mechanisms of zippered points, ultimately enhancing the scope and applicability of fixed point theory in abstract and applied mathematics.

Discussion

The integration of zippered point theorems and astringent mappings within generalized metric spaces marks a significant advancement in fixed point theory. Traditional fixed point results, such as Banach's contraction principle and its various generalizations, require strict contractive conditions and often assume a complete metric space with globally defined mappings. However, real-world problems and more abstract mathematical systems frequently operate in spaces where such assumptions do not hold. This necessitates the development of more adaptable theorems, such as those involving zippered points, which capture the essence of convergence under weaker conditions. The notion of astringent mappings emerges as a crucial relaxation of classical contractive mappings. These mappings do not necessarily reduce distances uniformly but exhibit convergence behavior under iterative application. Their application becomes especially valuable in generalized metric spaces like b-metric, G-metric, and partial metric spaces, which themselves extend the scope of standard metric spaces by altering triangle inequalities or redefining notions of distance. Within these spaces, sequences may behave differently, and convergence must be treated with greater nuance.

The concept of a zippered point acts as a bridge between convergence and fixed point existence. A zippered point, unlike a fixed point, may not immediately satisfy $T(x) = x$, but it anchors an iterative sequence whose terms get arbitrarily close to each other, allowing convergence to a fixed point under appropriate conditions. By combining this with astringent mappings, the current work expands the applicability of fixed point results to scenarios previously considered intractable. This new approach allows for the derivation of fixed point theorems under significantly relaxed assumptions, facilitating applications in areas such as differential equations, optimization theory, and dynamical systems—particularly in contexts where the underlying space does not exhibit strict metric properties. The generalization also permits flexibility in handling discontinuous mappings or mappings defined over non-complete spaces, provided certain convergence criteria are satisfied.

Moreover, the use of illustrative examples within this framework demonstrates the operational strength of the developed theorems. These examples verify that the conditions proposed are not merely theoretical artifacts but have real consequences for the existence and uniqueness of fixed points. At the same time, the inclusion of counterexamples reveals the precise boundaries within which the theorems remain valid, ensuring the mathematical rigor of the results. In conclusion, the unification of zippered point techniques with the behavior of astringent mappings in generalized metric spaces not only extends the theoretical landscape of fixed point theory but also opens new avenues for practical application. This synthesis provides a more flexible, robust analytical toolset for addressing a wider range of problems across both pure and applied mathematics.

Conclusion

The study presented a comprehensive investigation into the development of fixed point theorems by integrating the concepts of zippered points and astringent mappings within the framework of generalized metric spaces. This novel approach successfully broadens the scope of classical fixed point theory, which traditionally relies on strict contractive conditions and well-defined metric spaces. By relaxing these constraints through the use of astringent mappings and focusing on localized convergence properties inherent in zippered points, this work provides a more inclusive and powerful theoretical foundation. The results obtained demonstrate that even in the absence of global contractiveness or complete metric structures, fixed points can still be established under appropriate conditions. This contributes significantly to the field by addressing scenarios where traditional theorems fall short, particularly in mathematical models involving partial, b-metric, or G-metric spaces. The theoretical contributions are further validated by illustrative examples, reinforcing the practical utility and consistency of the proposed theorems.

Overall, this new approach not only enriches the existing body of fixed point literature but also offers promising directions for future research, including extensions to fuzzy, probabilistic, and modular spaces. It lays the groundwork for solving more complex problems in nonlinear analysis, optimization, and applied mathematical modeling, making it a valuable addition to both theoretical and applied disciplines.

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