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Cell : 9595 359 435, Ph No: 02172372010 Email: ayisrj@yahoo.in Website: www.isrj.net

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ALIGNED MAGNETIC FIELD ON CASSON FLOW WITH HEAT SOURCE BETWEEN INFINITE PARALLEL PLATES

C. K. Kirubhashankar and S. Ganesh

Department of Mathematics, Sathyabama University Rajiv Gandhi Salai, Chennai, India
Department of Mathematics, Sathyabama University Rajiv Gandhi Salai, Chennai, India

Abstract:-In the present study, we analyze the effect of heat source on Non-Newtonian fluid flow between infinite parallel plates in the presence of aligned magnetic field Casson fluid model is used to characterize the non-Newtonian fluid behavior. The coupled linear partial differential equations are solved by converting into ordinary linear differential equations by choosing the axial velocity, normal velocity and temperature field as a functions of y and t along with corresponding boundary conditions. The expressions are obtained for axial velocity, normal velocity and temperature field. The model has been analyzed to find the effects of various parameters such as, heat source parameter decay parameter, Casson parameter and Hartmann number on the axial velocity, temperature distribution.

Keywords: Parallel Plate Channel, Heat Source, Magnetic Field, Casson fluid.

INTRODUCTION

The magnetohydrodynamic flow between two parallel plates, known as Hartmann flow, is a classical problem that has many applications in magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Hartmann and Lazarus [1] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary, and insulated plates. Then, a lot of research work concerning the Hartmann flow has been obtained under different physical effects [2-12]. The rectangular channel problem has later been extended also to fluids obeying non-Newtonian constitutive equations.

Recently, due to increasing industrial and technological applications, the flows of non-Newtonian fluids attract much attention of researchers. For example, if one uses a non-Newtonian fluid as the coolant or heat exchangers, the required pumping power may be greatly reduced. Therefore, the fundamental analysis of the flow field of non-Newtonian fluids in a boundary layer adjacent to a stretching sheet or an extended surface is very important, and is an essential part in the study of fluid dynamics and heat transfer. Some materials, eg., melts, muds, condensed milk, glues, printing ink, emulsions, soaps, sugar solution, paints, shampoos, tomato paste, show the non-Newtonian properties of fluids. The features of non-Newtonian fluids are different from those of viscous fluids. Many investigations were made to examine flow over a stretching/shrinking sheet under different aspects of MHD, suction/injection, heat and mass transfer etc. [13–20].

Convective heat transfer plays a vital role during the handling and processing of non-Newtonian fluid flows. Mechanics of non-Newtonian fluid flows present a special challenge to engineers, physicists, and mathematicians. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids. In the process, a number of non-Newtonian fluid models have been proposed. Amongst these, the fluids of viscoelastic type have received much attention. In the literature, the vast majority of non-Newtonian fluid are concerned of the types, e.g., like the power law and grade two or three (Andersson and Dandapat [21], Hassanien [22], Sadeghy and Sharifi [23], Serdar and Salih Dokuz [24], Sajid et al. [25, 26]).

The purpose of this present work is to extend the flow and heat transfer analysis in boundary layer flow of a Casson fluid over an exponentially stretching sheet. Combined effects of suction/blowing and thermal radiation are investigated. Using transformations, a second order ordinary differential equation corresponding to the momentum equation and a second order differential equation corresponding to the heat equation are derived. The effects of velocity, temperature and concentration are studied for different parameters like heat source parameter decay parameter, Casson parameter and

Hartmann number.

MATHEMATICAL FORMULATION

Consider the flow of an incompressible viscous fluid past a flat sheet which coincides with the plane y =0. The fluid flow is confined to y> 0. It is also assumed that the rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as [27, 28].

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid as follows:

$$\tau_{ij} = \begin{cases} \left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}}\right) 2e_{ij}, & \pi > \pi_c, \\ \left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_c}}\right) 2e_{ij}, & \pi < \pi_c \end{cases}$$

where τ_{ij} is the (i, j)-th component of the stress tensor, $\pi = e_{ij} e_{ij}$ and e_{ij} are the (i, j)-th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, B is plastic dynamic viscosity of the non-Newtonian fluid , and py is the yield stress of the fluid.

The governing equations of such type of flow are, in the usual notations,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho}\sin^2\alpha + g\beta (T - T_0) \qquad (2)$$

$$0 = \frac{K'}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p}(T - T_0) \qquad (3)$$

where u and v as velocity components in the directions of x and y respectively (axial and normal respectively) at time t in the flow field. ν is the kinematic fluid viscosity, ρ is the fluid density, $\beta = \mu_B\sqrt{2\pi_c}/p_y$ is the Casson parameter, σ is the electrical conductivity of the fluid, and B_0 is the strength of magnetic field applied in the y direction. ρ and μ is the density and viscosity of the blood while p^* stands for pressure. K' is thermal conductivity; C_P is the specific heat at constant pressure. Q is the quantity of heat, T is the temperature and β is the volumetric expansion parameter while θ is the temperature distribution.

The boundary conditions are taken as:

$$\theta = e^{-\lambda^2 t},\ u = e^{-\lambda^2 t} \text{ at } y = -1$$
$$\theta = 0,\ u = 0 \text{ at } y = 1$$

Let us introduce the non-dimensional variables,

$$x^* = \frac{x}{h},\ y^* = \frac{y}{h},\ u^* = \frac{u}{m/2\mu h},\ v^* = \frac{v}{m/2\mu h}\ t^* = \frac{t}{h^2/\nu},$$

$$p^*(x,t) = \frac{dp/dx}{\rho (m/2\mu)^2 h^3},\ \theta^* = \frac{\theta}{\rho(m/2\mu)^2 h^3} \qquad (4)$$

Substituting equation (4) into equations (1) – (3), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (5)$$

$$\frac{\partial^2 u}{\partial y^2} - \left(\frac{\beta}{1+\beta}\right) u M^2 \sin^2\alpha = \left(\frac{1}{\mu}\right)\left(\frac{\beta}{1+\beta}\right)(p - g\theta) \quad (6)$$

$$\frac{\partial^2 \theta}{\partial y^2} + S\theta = 0 \quad (7)$$

where the heat source parameter, $S = \frac{Qh^2}{K_T}$

Analytical Solution of the Problem

With the above discussions in the previous section, let us choose the solutions of the equations (5) – (7) respectively as

$$u(y,t) = F(y)e^{-\lambda^2 t} \quad (8)$$

$$v(y,t) = G(y)e^{-\lambda^2 t} \quad (9)$$

$$\theta(y,t) = H(y)e^{-\lambda^2 t} \quad (10)$$

The boundary conditions are transformed to

$$H = 1,\quad F = 1,\quad \text{at } y = -1$$
$$H = 0,\quad F = 0,\quad \text{at } y = 1 \quad (11)$$

By virtue of (8) – (10), we obtain the equations (5) – (7) respectively as

$$F''(y) - \left(\frac{\beta}{1+\beta} M^2\sin^2\alpha\right) F(y) = \frac{\beta}{1+\beta}\left(\frac{p}{\mu} - g\lambda H(y)\right) \quad (12)$$

$$G = C \quad (13)$$

$$H''(y) + SH(y) = 0 \quad (14)$$

Solution of equation (14) using the boundary condition (11) is as follows

$$H(y) = \frac{1}{2}\left(\frac{1}{\cos\sqrt{S}}\cos\left(\sqrt{S}\,y\right) - \frac{1}{\sin\sqrt{S}}\sin\left(\sqrt{S}\,y\right)\right) \quad (15)$$

From (10) and (15) the temperature distribution is given by

$$\theta(y,t) = \frac{1}{2}\left(\frac{1}{\cos\sqrt{S}}\cos\left(\sqrt{S}\,y\right) - \frac{1}{\sin\sqrt{S}}\sin\left(\sqrt{S}\,y\right)\right)e^{-\lambda^2 t} \quad (16)$$

Using the equation (16) into equation (12) we get

$$F''(y) - \frac{\beta}{1+\beta}M^2\sin^2\alpha\, F(y) = \frac{\beta}{1+\beta}\left(\frac{p}{\mu} - \frac{g\lambda}{2}\left(\frac{1}{\cos\sqrt{S}}\cos\left(\sqrt{S}\,y\right) - \frac{1}{\sin\sqrt{S}}\sin\left(\sqrt{S}\,y\right)\right)\right) \quad (17)$$

From equation (8) and (16) the velocity of the flow of the fluid parallel to the direction of the channel is obtained as,

$$u(y,t) = \left(c_1e^{\xi y} + c_2e^{-\xi y} - \frac{p}{\mu M^*} + c_3\left(\frac{\cos\sqrt{S}\,y}{\cos\sqrt{S}} - \frac{\sin\sqrt{S}\,y}{\sin\sqrt{S}}\right)\right)e^{-\lambda^2 t} \quad (18)$$

where $p_1 = \frac{p^*}{e^{-\lambda^2 t}}$, $\xi = \sqrt{\frac{\beta}{1+\beta}}M^*$, $M^{*2} = M^2\sin^2\alpha$,

$$c_1 \mid \frac{1}{4}\left|\left|1 \mid \frac{c_3}{2}\right|\right|\frac{1}{\cos|} \mid \frac{1}{\sin|}\left|\mid \frac{2p}{|M^* \cos|}\right|, \; c_2 \mid \left|\frac{1}{4}\right|1 \mid \frac{c_3}{2}\left|\right|\frac{1}{\cos|} \mid \frac{1}{\sin|}\left|\mid \frac{2p}{|M^* \cos|}\right|,$$

$$c_3 \mid \left|\frac{|}{1 \,|\, |}\right|\frac{g|}{2(S \,|\, |^2)}$$

From equations (9) and (13), the velocity of the fluid flow perpendicular to the direction of the channel is given by

$$v(y,t) \mid Ce^{||^2 t} \qquad (19)$$

where C is an arbitrary constant.

Equations (16), (18) and (19) show the temperature distribution, the axial velocity and normal velocity respectively.

RESULTS AND DISCUSSIONS

To study the behavior of the velocity and temperature profiles, curves are drawn for various values of the parameters that describe the flow. The results obtained for the steady flow are displayed in Figures 2-13. It should be mentioned that the results obtained here in reduce to those reported by Om Prakash when = /2 and tends to infinity which gives validity of the present solution.

Effects of different physical parameters on temperature fields

Figure 1 shows the performance of temperature distributions versus y at Q = 0.25,h = 1, K = 1, λ = 0.25 and t = 1 for different values of heat source parameter (S = 0.25, 0.5, 0.75, 1, 1.25). We observe that the temperature field increases with increasing the values of S, temperature at the lower plate is higher than the temperature at the upper plate.

Figure 2 emphasizes that the temperature field distribution for different values of heat source and decay parameter (S = 0.25, 0.5, 0.75, 1, 1.25) at Q = 0.25,h = 1, K = 1, λ = 0.25 and t = 1. The effect of heat source and decay parameter on temperature steadily decreases with increasing the values of heat source and decay parameter.

It is clear from Figure 3 that temperature field distribution increases with negatively correlated values of the heat source and decay parameter , at Q = 0.25,h = 1, K = 1, λ = 1.25 and t = 1.

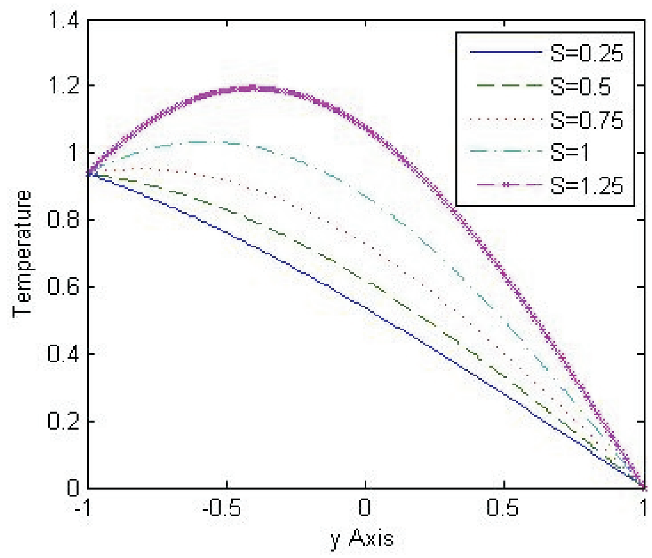

Figure 1. Temperature Field for different values
of values of Heat Source Parameter (S)

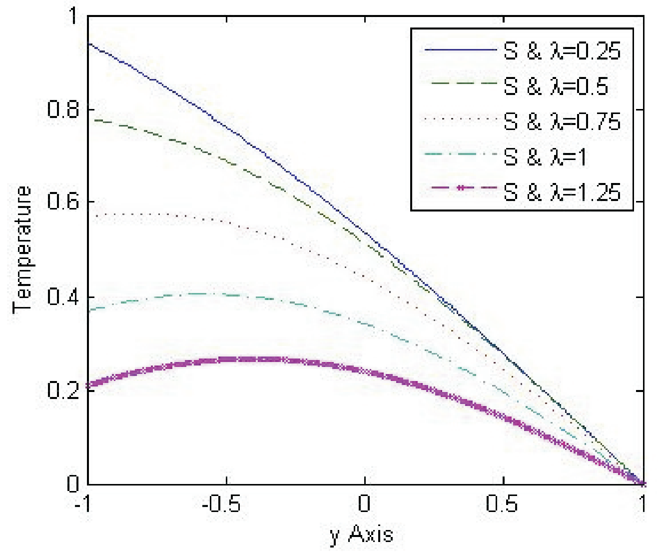

Figure 2. Temperature Field for different values
of values of Heat Source Parameter (S) and Decay parameter (λ)

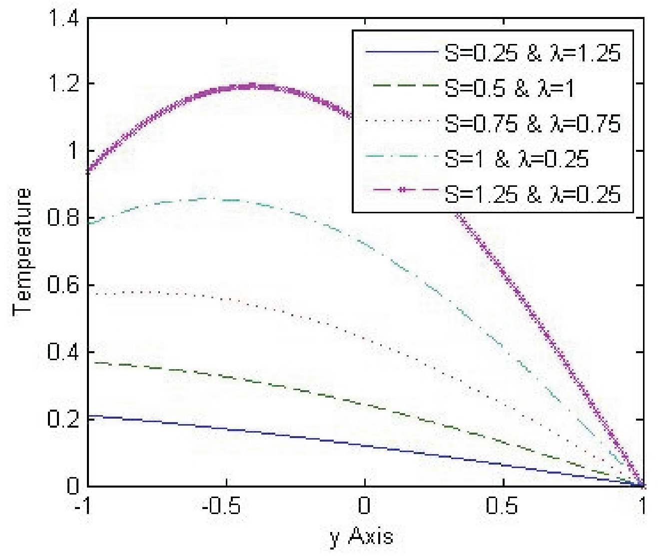

Figure 3. Temperature Field for different values of values
of increasing Heat Source Parameter (S) and decreasing Decay parameter (λ)

Effects of different physical parameters on velocity fields

Figure 4 exhibits the axial velocity profiles for several values of heat source parameter (S = 3, 3.25, 3.5, 3.75,4) at

$Q = 0.75,\ h = 1, K = 0.25,\ | = 0.25\ \ M = 1,\ | = 0.25, | = |/12, | = 0.25, | = 1, g = 9.8, p = 0.5$ and $t = 1$.

It is observed that axial velocity increases with increasing the heart source parameter S.
Figure 5 indicates the effect of magnetic field on the axial velocity for different values of Hartmann number

(M = 1, 1.25, 1.5, 1.75, 2) at $Q = 0.75, h = 1, K = 0.25, | = 0.25\ \ M = 1, | = 0.25, | = |/6, | = 0.25, | = 1, g = 9.8,$

p = 0.5 and t = 1. It is observed that the axial velocity decreases with increasing the magnetic field.
Figure 5 indicates the effect of magnetic field on the axial velocity for different values of Hartmann number (M = 1, 1.25, 1.5, 1.75, 2) at $Q = 0.75, h = 1, K = 0.25, | = 0.25\ \ M = 1, | = 0.25, | = |/6, | = 0.25, | = 1, g = 9.8, p = 0.5$

and t = 1. It is observed that the axial velocity decreases with increasing the magnetic field.
Figure 4 shows the influence of inclination angle on the velocity profiles. As the angle of inclination increases causes the decrease of velocity.
Figure 8 defines the effect of decay parameter on the axial velocity for different values of decay parameter
(| = 0.5, 0.75, 1, 1.25, 1.5) at Q = 0.25, h = 1, K = 0.25, | = 0.25 M = 2, | = 0.25, | = |/2, | = 0.25, | = 1, g = 9.8,

p = 0.5 and t = 1. It is observed that the axial velocity increases with increasing the decay parameter up to y -0.4 and for y -0.3 axial velocity decreases with increasing the decay parameter.
Effects of Casson parameter on velocity profiles for unsteady motion are clearly exhibited in Figure 8 the behavior of velocity with increasing is noted at
Q = 0.75, h = 1, K = 0.25, | = 0.25 M = 1, | = 0.25, | = |/6, | = 0.25, | = 1, g = 9.8, p = 0.25 and t = 1.
It is observed that the axial velocity is found to decrease with increasing Casson parameter .β.

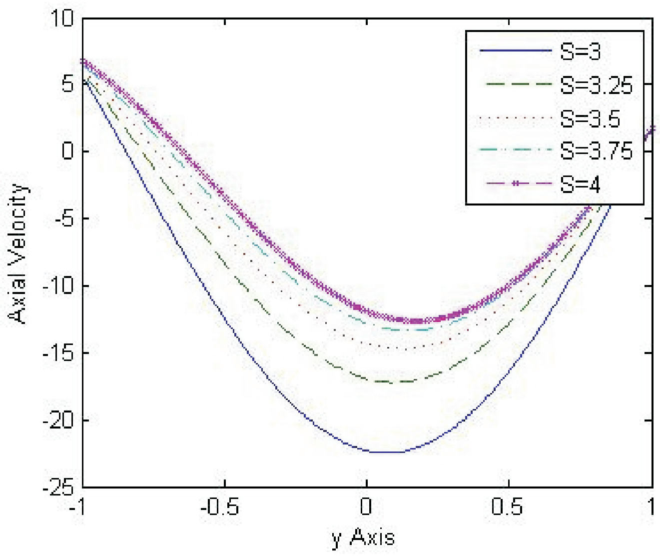

Figure 4. Axial velocity for different values of values of Heat Source Parameter (S)

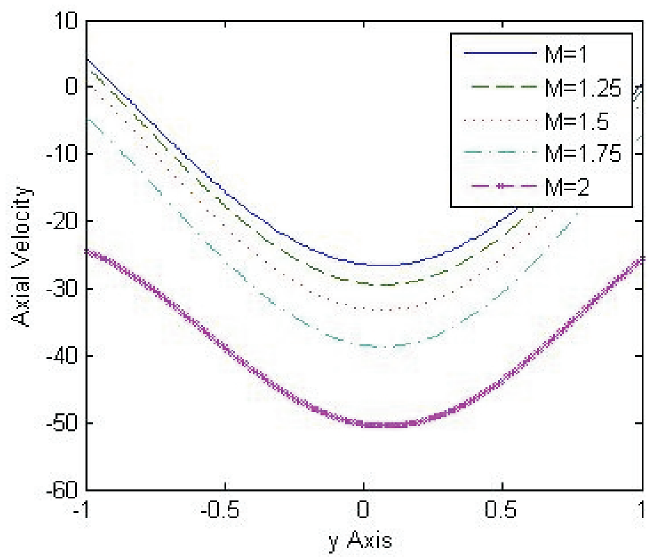

Figure 5. Axial velocity for different values of values of Magnetic Field Parameter (M)

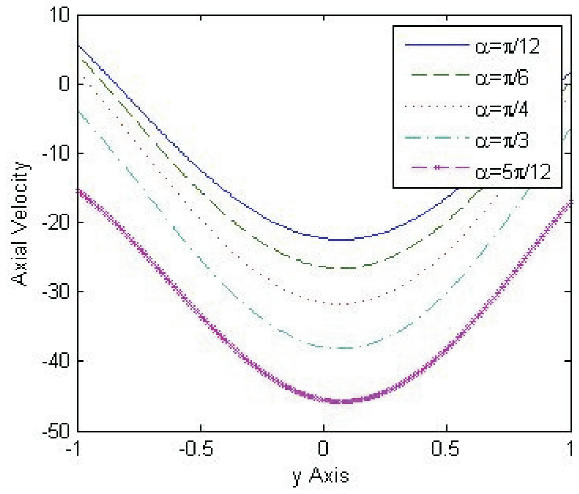

Figure 6. Axial velocity for different values of inclined angle (α)

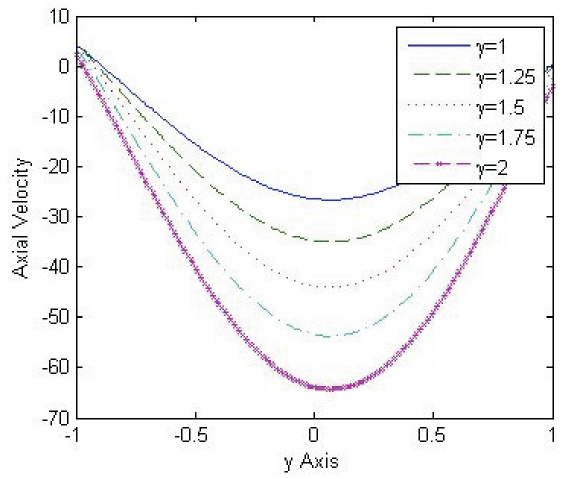

Figure 7. Axial velocity for different values of values of Casson Parameter (λ)

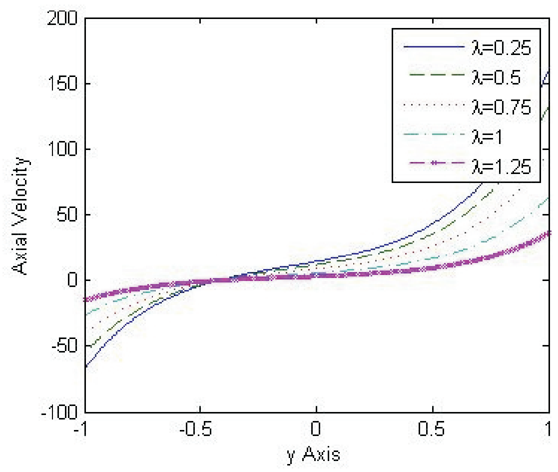

Figure 8. Axial velocity for different values of Decay Parameter (λ).

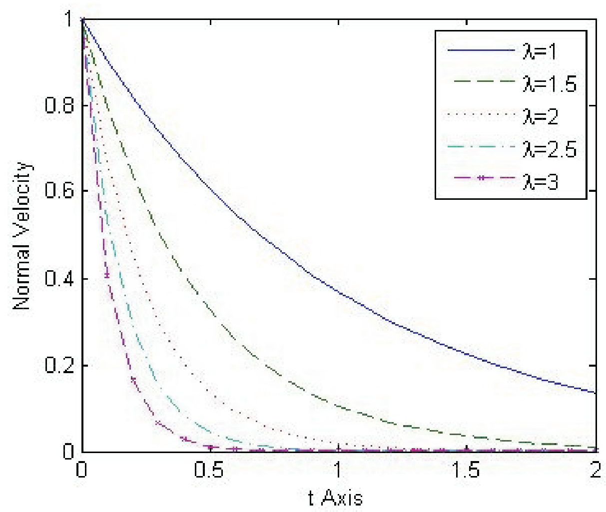

Figure 9. Normal velocity for different values of Decay Parameter (λ).

CONCLUSION

The MHD flow of Casson fluid and heat transfer over a stretching sheet are investigated taking into consideration the thermal radiation effect. From the study, the following remarks can be summarized

The temperature field increases with increasing the heat source parameter (S). Temperature field decreases for positively correlated values of the heat source parameter (S) and decay parameter () and temperature field increases for negatively correlated values of the the heat source parameter (S) and decay parameter (). The axial velocity increases with increasing the heart source parameter (S). Increase in inclination angle results in a decrease in the velocity. The axial velocity decreases with increasing the magnetic field (Ha) and Casson parameter (). The effect of increasing values of the Casson parameter is to suppress the velocity field. It is noticed that when the fluid parameter approaches infinity, the problem in the given case reduces to a Newtonian case. The normal velocity decreases with increasing the decay parameter and tending to zero very fast for higher values of the decay parameter. The results may be helpful for possible technological applications in liquid-based systems involving stretchable materials.

REFERENCES

1.Hartmann J. and Lazarus F. (1937): Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 15 (6,7).
2.TAO, I. N. (1960): 'Magnetohydrodynamic effects on the formation of Couette flow'. J. of Aerospace Sci. 27, 334.
3.Alpher R. A. (1961): 'Heat transfer in magnetohydrodynamic flow between parallel plates'. Int. J. Heat and Mass Transfer. 3, 108.
4.Sutton G. W. and Sherman A. (1965): Engineering Magnetohydrodynamics. McGraw-Hill Book Co.
5.Cramer K. R. and Pai, S.-I. (1973): Magnetofluid dynamics for engineers and applied physicists. McGraw-Hill Book Co.
6.Nigam S. D. and Singh, S.N. (1960): Quart. J. Mech. Appl. Math. 13, 85.
7.Tani, I. (1962): 'Steady motion of conducting fluids in channels under transverse magnetic fields with consideration of Hall effect'. J. of Aerospace Sci. 29, 287.
8.Soundalgekar V. M., Vignesam N. V., and Takhar H. S. (1979): 'Hall and Ion-slip effects in MHD Couettee flow with heat transfer'. IEEE Trans. Plasma Sci. PS-7 (3), 178.
9.Soudalgekar V. M. and Uplekar A. G. (1986): IEEE Trans. Plasma Sci. PS-14 (5), 579.
10.Attia H. A. (1999): 'Transient MHD flow and heat transfer between two parallel plates with temperature dependent viscosity'. Mech. Res. Comm., 26 (1), 115.
11.Ganesh S, Krishnambal S,(2006), 'Magnetohydrodynamic flow of viscous fluid between two parallel porous plates', Journal of Applied Sciences, Asian Network for scientific information, 6(11):2450-2425,2006.
12.Mohamed Ismail A, Ganesh S,(2014), ' Unsteady Stokes Flow of Dusty Fluid between Two Parallel Plates through Porous Medium',Applied Mathematical Sciences, Vol. 8, 2014, no. 5, 241 – 249.
13.Ariel, P.D., Hayat, T. and Ashgar, S., (2006),'The flow of a elasto-viscous fluid past a stretching sheet with partial slip', Acta Mech., 187, pp. 29–35.
14.Nadeem, S., Hussain, A., Malik, M.Y. and Hayat, T., (2009), 'Series solutions for the stagnation flow of a second-grade fluid

over a shrinking sheet', Appl. Math. Mech., 30(10), pp. 1255–1262.
15.Nadeem, S., Hussain, A. and Khan, M., (2010), 'Stagnation flow of a Jeffrey fluid over a shrinking sheet', Z. Naturforsch., 65a, pp. 540–548.
16.Hayat, T. and Qasim, M., (2010), 'Radiation and magnetic field effects on the unsteady mixed convection flow of a second grade fluid over a vertical stretching sheet', Int. J. Numer. Methods Fluids, 66(7), pp. 820–832.
17.Nadeem, S. and Faraz, N., (2010), 'Thin film flow of a second grade fluid over a stretching/shrinking sheet with variable temperature-dependent viscosity', Chin. Phys. Lett., 27(3), p. 034704.
18.Ishak, A., Nazar, R. and Pop, I., (2009), 'Heat transfer over an unsteady stretching permeable surface with prescribed wall temperature', Nonlinear Anal. RWA, 10, pp. 2909–2913.
19.Hayat, T., Shehzad, S.A., Qasim, M. and Obaidat, S., (2011), 'Steady flow of Maxwell fluid with convective boundary conditions', Z. Naturforsch., 66a, pp. 417–422.
20.Wang, C. and Pop, I., (2006), 'Analysis of the flow of a power-law fluid film on an unsteady stretching surface by means of homotopy analysis method', J. Non-Newtonian Fluid Mech., 138, pp. 161–172.
21.Andersson HI, Dandapat BS., 'Flow of a power law fluid over a stretching sheet'. Appl Anal Continuous Media 1992; 1:339–47.
22.Hassanien IA. (1996), 'Flow and heat transfer on a continuous flat surface moving in a parallel free stream of power-law fluid'. Appl Model; 20:779–84.
23.Sadeghy K, Sharifi M. (2004) 'Local similarity solution for the flow of a 'second-grade' viscoelastic fluid above a moving plate'. Int J Nonlinear Mech; 39:1265–73.
24.Serdar B, Salih Dokuz M. (2006) 'Three-dimensional stagnation point flow of a second grade fluid towards a moving plate'. Int J Eng Sci;44:49–58.
25.Sajid M, Hayat T, Asgharm S. (2007) 'Non-similar analytic solution for MHD flow and heat transfer in a third-order fluid over a stretching sheet'. Int J Heat Mass Transfer; 50:1723–36.
26.Sajid M, Ahmad I, Hayat T, Ayub M. (2009) 'Unsteady flow and heat transfer of a second grade fluid over a stretching sheet'. Commun Nonlinear Sci Num Simul; 14:96–108.
27.K. Bhattacharyya, T. Hayat, and A. Alsaedi, (2013) “Exact solution forboundary layer flow of Casson fluid over a permeable stretching/shrinking sheet,” Zeitschrift f¨ur Angewandte Mathematik und Mechanik,.
28.M. Nakamura and T. Sawada, (1988) “Numerical study on the flow of a non-Newtonian fluid through an axisymmetric stenosis,” Journal of Biomechanical Engineering, vol. 110, no. 2, pp. 137–143.

C. K. Kirubhashankar
Department of Mathematics, Sathyabama University Rajiv Gandhi Salai, Chennai, India