Indian Streams Research Journal
ISSN 2230-7850
Impact Factor : 2.1506(UIF)
Volume-4 | Issue-2 | March-2014 Available online at www. isrj.net

STUDIES ON MUTUAL INFORMATION WITH APPLICATION TO FIQM

Rahul Kumar[1] and R.S.Roy [2]

[1]Assistant Professor Dept. Of Physics Dr.b.n.y.degree College, Kachnar (Saran)
[2] Principal Ram Jaipal College, Chapra

ABSTRACT : This paper provides the three popular correlation-based FIQMs. Closed form expressions for their quality estimates which has been discussed on the basis of employing the technique for mutual information.

Key-words: Mutual information, Information, & FIQMs

INTRODUCTION

Mutual information is a quantity in information theory which is used to measure the amount of information that one random variable contains about another random variable [1-3]. In this paper, we have discussed about the mutual information in two random variables using a technique known as FIQM.

MATHEMATICAL ASPECTS

The mutual information of two continuous random variables *x* and *y* is defined as:

$$F_{MI}(x,y) = \int_y \int_x p(x,y) \log_2 \frac{p(x,y)}{p_1(x)p_2(y)} dxdy, \tag{1}$$

where $p(x, y)$ is the joint probability density function of x and y, and $p_1(x)$ and $p_2(y)$ are the marginal probability density functions of x and y respectively. Assuming that x, y are Gaussian random variables, $FMI(x, y)$ can be further expressed as:

$$F_{MI}(x,y) = \frac{1}{2} \log_2 \left(\frac{1}{1 - \left(\frac{\sigma_{x,y}}{\sigma_x \sigma_y} \right)^2} \right), \tag{2}$$

where $\frac{\sigma_{x,y}}{\sigma_x \sigma_y}$ is the correlation coefficient between *x* and *y*.

Substituting $F_{MI}(x, y)$ for F in (2), $F_{MI}(x, y)$ was employed as the correlation metric in [3] to evaluate the amount of information transferred from the source images to the fused image. From equations (1) and (2) we can easily tell that f, z_1 and z_2 are Gaussian random variables and we can calculate the mutual information based FIQM.

$$Q_{MI} = \frac{1}{2}\log_2\left(\frac{\left(\left(w_1\beta_1 + w_2\beta_2\right)^2 \sigma_s^2 + w_1^2\sigma_1^2 + w_2^2\sigma_2^2\right)^2}{w_1^2 w_2^2 \left(\beta_1^2\sigma_s^2\sigma_2^2 + \beta_2^2\sigma_s^2\sigma_1^2 + \sigma_1^2\sigma_2^2\right)^2}\right) + \frac{1}{2}\log_2\left(\beta_1^2\sigma_s^2 + \sigma_1^2\right) + \frac{1}{2}\log_2\left(\beta_2^2\sigma_s^2 + \sigma_2^2\right). \quad (3)$$

UIQI BASED FIQM

To evaluate the similarity of two images the metric out performs the traditional mean squared error (MSE) based image quality metric because it considers the structural distortions of source images including loss of correlation, luminance distortion and contrast distortion .To calculate the UIQI, we use

$$F_{UIQI}(x, y) = \frac{4\sigma_{x,y}\bar{x}\bar{y}}{(\sigma_x^2 + \sigma_y^2)(\bar{x}^2 + \bar{y}^2)} \quad (4)$$

where x, σ^2_x, y, σ^2_y denote the mean values and variances of *x* and *y* respectively and $\sigma_{x,y}$ means the covariance between *x* and *y*.

To assess the performance of image fusion algorithms, we calculate the quantities in (4) we can obtain the closed-form expression of UIQI based image fusion FIQM

$$Q_{UIQI} = \frac{\left(\left(w_1\beta_1 + w_2\beta_2\right)\beta_1\sigma_s^2 + w_1\sigma_1^2\right)\left(w_1\beta_1 + w_2\beta_2\right)\beta_1}{\left(\left(w_1\beta_1 + w_2\beta_2\right)^2\sigma_s^2 + w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \beta_1^2\sigma_s^2 + \sigma_1^2\right)\left(\left(w_1\beta_1 + w_2\beta_2\right)^2 + \beta_1^2\right)} + \frac{\left(\left(w_1\beta_1 + w_2\beta_2\right)\beta_2\sigma_s^2 + w_2\sigma_2^2\right)\left(w_1\beta_1 + w_2\beta_2\right)\beta_2}{\left(\left(w_1\beta_1 + w_2\beta_2\right)^2\sigma_s^2 + w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \beta_2^2\sigma_s^2 + \sigma_2^2\right)\left(\left(w_1\beta_1 + w_2\beta_2\right)^2 + \beta_2^2\right)}. \quad (5)$$

EDGE BASED FIQM

Considering the fact that the human visual system (HVS) is quite sensitive to the edge information in an image,

$$F_{UIQI}(x', y'), \quad (6)$$

where *x′* and *y′* denote the edge images corresponding to *x* and *y*, respectively. The “edge image” is defined as the Euclidean norm of the horizontal and vertical gradient edge information which can be achieved by applying a horizontal gradient convolution mask and a vertical gradient convolution mask to the original image respectively.

To derive the closed-form expression of Q_E,

$$Q_E = \frac{\left(\left(w_1\beta_1 + w_2\beta_2\right)\beta_1\sigma_s'^2 + w_1\sigma_1'^2\right)\left(w_1\beta_1 + w_2\beta_2\right)\beta_1}{\left(\left(w_1\beta_1 + w_2\beta_2\right)^2\sigma_s'^2 + w_1^2\sigma_1'^2 + w_2^2\sigma_2'^2 + \beta_1^2\sigma_s'^2 + \sigma_1'^2\right)\left(\left(w_1\beta_1 + w_2\beta_2\right)^2 + \beta_1^2\right)} + \frac{\left(\left(w_1\beta_1 + w_2\beta_2\right)\beta_2\sigma_s'^2 + w_2\sigma_2'^2\right)\left(w_1\beta_1 + w_2\beta_2\right)\beta_2}{\left(\left(w_1\beta_1 + w_2\beta_2\right)^2\sigma_s'^2 + w_1^2\sigma_1'^2 + w_2^2\sigma_2'^2 + \beta_2^2\sigma_s'^2 + \sigma_2'^2\right)\left(\left(w_1\beta_1 + w_2\beta_2\right)^2 + \beta_2^2\right)} \quad (7)$$

by taking the steps explained to calculate Q_{UIQI}. These three FIQMs are direct functions of the correlation coefficient.

DISCUSSION

In this section we explain intuitively the poor behavior of the aforementioned three correlation-based image fusion quality measures. Let us start with the mutual information based quality measure Q_{MI}.

According to equation (7), $F_{MI}(f, z_k)$ (k = 1, 2) can be calculated as:

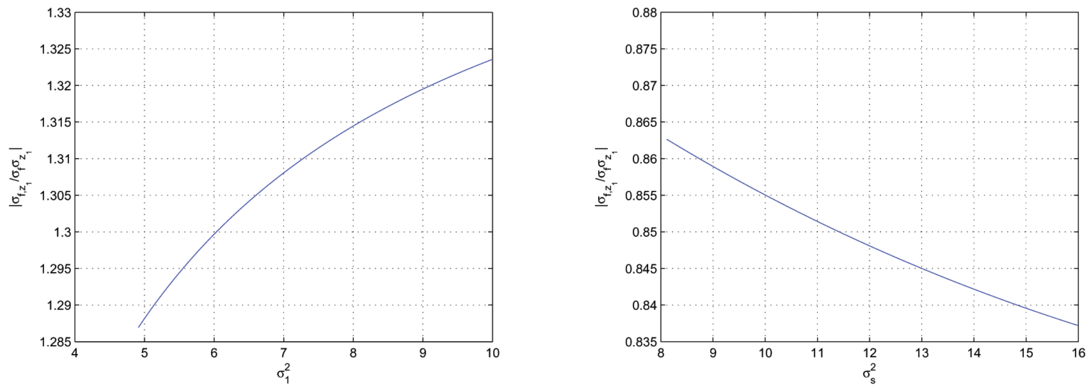
$$F_{MI}(f, z_k) = \frac{1}{2}\log_2\left(\frac{1}{1-\left(\frac{\sigma_{f,z_k}}{\sigma_f\sigma_{z_k}}\right)^2}\right) \quad (8)$$

where $\frac{\sigma_f z_k}{\sigma_f \sigma z_k}$ is the correlation coefficient between f and z_k. Apparently $\left|\frac{\sigma_f z_k}{\sigma_f \sigma z_k}\right| \in (0, 1)$, so we have that $F_{MI}(f, z_k)$ is monotonically increasing with $\left|\frac{\sigma_f z_k}{\sigma_f \sigma z_k}\right|$ which can be written as:

$$\left|\frac{\sigma_{f,z_k}}{\sigma_f\sigma_{z_k}}\right| = \frac{|\beta_k(w_1\beta_1 + w_2\beta_2)\sigma_s^2 + w_k\sigma_k^2|}{\sqrt{\beta_k^2\sigma_s^2 + \sigma_k^2}\sqrt{(w_1\beta_1 + w_2\beta_2)^2\sigma_s^2 + w_1^2\sigma_1^2 + w_2^2\sigma_2^2}}, k = 1, 2. \quad (9)$$

Since $F_{MI}(f, z_1)$, the first term of Q_{MI} in (2.5), depends on $\left|\frac{\sigma_f z_1}{\sigma_f \sigma z_1}\right|$ and $F_{MI}(f, z_2)$, the second term of Q_{MI} in (5), is determined by $\sigma f \left|\frac{\sigma_f z_2}{\sigma_f \sigma z_2}\right|$ we can discuss how Q_{MI} changes with $\sigma^2{}_1$ and $\sigma^2{}_s$ by just analyzing $\left|\frac{\sigma_f z_1}{\sigma_f \sigma z_1}\right|$ and $\left|\frac{\sigma_f z_2}{\sigma_f \sigma z_2}\right|$

THE CHANGE IN *QMI* WITH INCREASING *Σ*2



(a) $\left|\frac{\sigma_{f,z_1}}{\sigma_f \sigma_{z_1}}\right|$ increases with increasing σ_1^2, for $w_1 = 0.9501; w_2 = 0.0499; \beta_1 = 0.2311; \beta_2 = 0.7689; \sigma_s^2 = 6.0684; \sigma_2^2 = 8.913$.

(b) $\left|\frac{\sigma_{f,z_1}}{\sigma_f \sigma_{z_1}}\right|$ decreases with increasing σ_s^2, for $w_1 = 0.6412; w_2 = 0.3588; \beta_1 = 0.0162; \beta_2 = 0.8369; \sigma_1^2 = 6.9778; \sigma_2^2 = 4.6189$.

Figure 1: $|\frac{\sigma_f Z_1}{\sigma_f \sigma Z_1}|$ increases with increasing $\sigma^2{}_1$ and decreases with increasing $\sigma^2{}_s$.

When $\sigma^2{}_1$ increases, $|\frac{\sigma_f Z_2}{\sigma_f \sigma Z_2}|$ always decreases because its numerator doesn't change and its denominator keeps increasing. On the other hand, from equation (8) we can see that the numerator and denominator of $|\frac{\sigma_f Z_1}{\sigma_f \sigma Z_1}|$ both increase when $\sigma^2{}_1$ increases. If the numerator part dominates, then $|\frac{\sigma_f Z_1}{\sigma_f \sigma Z_1}|$ increases with increasing $\sigma^2{}_1$ which is shown in Fig. 1(a). Now when the first term in (3), $F_{MI}(f, z_1)$, increases faster than the second term, $F_{MI}(f, z_2)$, decreases, then we can conclude that Q_{MI} increases with increasing noise power $\sigma^2{}_1$.

THE CHANGE IN Q_{MI} WITH INCREASING $\Sigma^2{}_S$

When $\sigma^2{}_s$ increases, the numerator and denominator of $|\frac{\sigma_f Z_k}{\sigma_f \sigma Z_k}|$ both increase. If the denominator part increases faster, then $|\frac{\sigma_f Z_k}{\sigma_f \sigma Z_k}|$ decreases as shown in Fig.1(b). When a decreasing term in (3) dominates, Q_{MI} decreases with increasing $\sigma^2{}_s$. Some sufficient conditions are omitted due to space limits.

For the UIQI based FIQM, equation (8) can be expanded as:

$$F_{UIQI}(f, z_k) = \frac{4\sigma_{f,z_k}}{\sigma_f^2 + \sigma_{z_k}^2} \frac{(w_1\beta_1 + w_2\beta_2)\beta_k}{(w_1\beta_1 + w_2\beta_2)^2 + \beta_k^2}, k = 1, 2. \quad (10)$$

We found that $F_{UIQI}(f, z_k)$ is monotonically increasing with a similar quantity $\frac{\beta_k \sigma_{f,z_k}}{\sigma_f^2 + \sigma_{z_k}^2}$ which increases with the noise power and decreases with the power of the signal of interest in some cases. Therefore the poor behavior of the UIQI based FIQM can be explained similarly. The same explanation also applies to the edge based FIQM because *QE* and Q_{UIQI} have almost the same closed-form expression.

CONCLUSION

In conclusion, this paper gives the idea of mutual information based on numerical analysis.

REFERENCES

[1] E. D. Dickmanns and B. D. Mysliwetz, "Recursive 3-D road and relative ego-state recognition," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 14, no. 2, pp. 199–213, Feb. 1992.

[2] E. D. Dickmanns, S. Werner, S. Kraus, and R. Schell, "Experimental results in autonomous landing approaches by dynamic machine vision," in *Proceedings of SPIE - The International Society for Optical Engineering*, volume 2220, pp. 304– 313, Orlando, FL, USA, 1994.

[3] O. E. Drummond, "Methodologies for performance evaluation of multitarget multisensor tracking," in *Proceedings of SPIE - The International Society for Optical Engineering*, volume 3809, pp. 355–369, Denver, CO, USA, 1999.