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ON Zc SEPARATION AXIOMS AND *g.Zc* – CLOSED , *Zc.g* – CLOSED SETS.

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ABSTRACT:
In this paper ,a new kind of sets called Zc-generalized closed sets and generalized Zc-closed sets are introduced and studied in a topological spaces by using the concept of operation on topological space. Also Zc-separation axioms are introduced and discussed, and we also studied the basic characterizations of the Zc-regular, Zc-normal spaces.

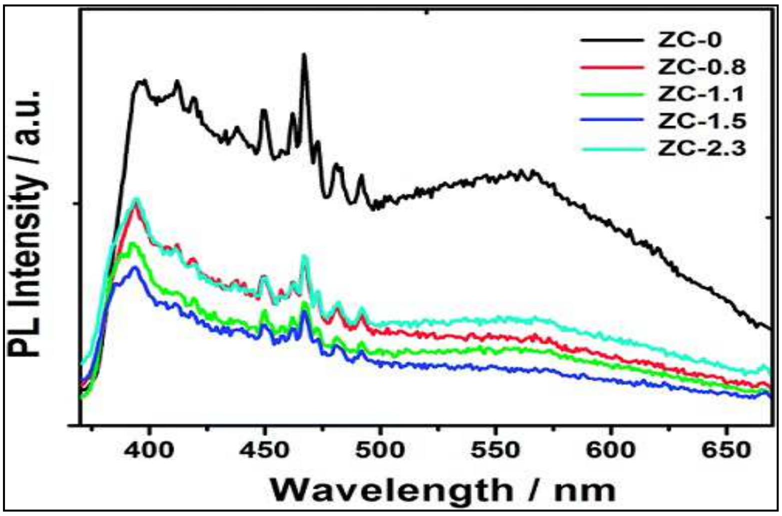
Keywords: *Zc-open, Zc-limit point, Zc-exterior, Zc-regular, Zc-normal, Zc*-Regular, Almost Zc*-Regular, Zc.g-closed, g.Zc-closed.*

1.INTRODUCTION:

The concept of *Z*-open sets was introduced first by A.I.EL-Magharabi and A.M.Mubarki [1] in 2011. Throughout this paper (X,τ) or simply X represent topological space with topology τ. The end or omission of the theorem,proposition or lemma is denoted by ■.

2.PRELIMINARIES:

Definition 2.1 [2] : A subset A of a space X is said to be



i) Z-open set if $A \subseteq cl(\delta\text{-}int(A)) \cup int(cl(A))$,
ii) Z-closed set if $int(\delta\text{-}cl(A)) \cap cl(int(A)) \subseteq A$. The family of all Z-open (resp. Z-closed sets) subsets of a space (X,τ) will be denoted by $ZO(X)$ (resp., $ZC(X)$).

Definition 2.2[4]: (i) A subset A of a space X is Zc-open`if for each $x \in A \in ZO(X)$,there exists a closed set F such that $x \in F \subset A$. A subset A of a space X is Zc-closed if $X - A$ is Zc-open. The family of all Zc-open(resp. Zc-closed) subsets of a topological space (X,τ) is denoted by $ZcO\ (X,\tau)$ or $ZcO(X)$ (resp. $ZcC\ (X,\tau)$ or $ZcC(X)$).
(ii) A subset A of a space X is Zs-open if for each $x \in A \in ZO(X)$,there exists a semiclosed set F such that $x \in F \subset A$. A subset A of a space X is Zs-closed if $X - A$ is Zs-open. The family of all Zs-open subsets of a topological space (X,τ) is denoted by $ZsO(X,\tau)$ or $ZsO(X)$.

Definition 2.3[2]: Let A be a subset of a topological space (X,τ).Then a point $p \in X$ is called a Z -limit point of a set $A \subseteq X$ if every Z-open set $G \subseteq X$ containing p contains a point of A other than p. The set of all Z-limit points of A is called a Z-derived set of A and is denoted by $Z\text{-}d(A)$.

Definition 2.4[5]: A subset A of a topological space (X,τ) is said to be (i)δ-closed if $A = \delta\text{-}\ cl(A)$,where $\delta\text{-}cl(A) = \{x \in X : A \cap int(cl(U)) \neq \emptyset,\ U \in \tau$ and $x \in \tau\}$and its complement is δ-open.
(ii)θ-open if for each $x \in A$, there exists an open set G such that $x \in G \subset cl(G) \subset A$ and its complement is θ-closed.

Definition 2.5:[3] Let A be a subset of a topological space (X,τ). A is said to be θ-regular open in X iff $A=\theta$-int(θ-cl(A)).

Definition 2.6:[1] A space X is called θ-T_2 space iff for each $x \neq y$ in X there exist disjoint θ-open sets U,V such that $x \in U$, $y \in V$.

3.BASIC PROPERTIES OF *ZC*-OPEN SETS:

Definition 3.1: Let (X,τ) be a topological space. Then

(i) the union of all Zc-open sets contained in A is called the *Zc-interior* of A and is denoted by Zc -$Int(A)$.
(ii) the intersection of all Zc-closed sets containing A is called the *Zc-closure* of A and is denoted by Zc-$cl(A)$.

Result 3.2: If A is Zc-closed then Zc-$cl(A) = A$.

Propositon 3.3: Let A be an Zc-open subset of X. Then $x \in Zc$-$cl(A)$ iff for any Zc-open set U containing x, $U \cap A \neq \emptyset$.

Proof: Let $x \in Zc$-$cl(A)$.Suppose that $U \cap A=\emptyset$.Suppose that $U \cap A=\emptyset$ for some Zc-open set U which contains x. Then $(X-U)$ is a Zc-closed set and $A \subseteq (X-U)$ and so $Zccl(A) \subseteq Zc$-$cl(X-U)$.Thus Zc-$cl(A) \subseteq X-U$, by result 3.2 and so $x \in (X-U)$ which is a contradiction. Hence $U \cap A \neq \emptyset$.Conversely, Suppose that there exists an Zc-openset U containing x with $U \cap A=\emptyset$.Then $A \subseteq (X-U)$ and $(X-U)$ is Zc-closed with $x \notin (X-U)$.Hence $x \notin Zc$-$cl(A)$, a contradiction ■.

Definition 3.4: Let (X,τ) be a topological space. Then

(i) the union of all Zs-open sets contained in A is called the *Zs-interior* of A and is denoted by Zs- $Int(A)$.
(ii) the intersection of all Zs-closed sets containing A is called the *Zs-closure* of A and is denoted by Zs- $cl(A)$.

Proposition 3.5: Let A be any subset of a space X. If a point x is in the Zs- $Int(A)$,then there exists a semi closed set F of X containing x such that $F \subseteq A$.

Proof: Suppose that $x \in Zs$ -$Int(A)$,then there exists a Zs-open set U of X containing x such that $U \subseteq A$. Since U is a Zs-open, by definition there exists a semi closed set F containing x such that F is the union of semiclosed and Z-open by proposition 4.3 of [2].

Hence $x \in F \subseteq A$ ■.

Theorem 3.6: Let A be a subset of a space X. Then

(i) $X \setminus Zc$ -$Int(A) = Zc$-$cl(X \setminus A)$
(ii) $X \setminus Zc$-$cl(A) = Zc$ -$Int(X \setminus A)$

Proof: For (i): For any point $x \in X$,if $x \in X \setminus Zc$- $Int(A) \Leftrightarrow x \notin Zc$- $Int(A) \Leftrightarrow$for each $B \in ZcO(X)$ containing x, we have $A \cap B=\emptyset \Leftrightarrow x \in B \subseteq (X \setminus A) \Leftrightarrow x \in Zc$-$cl$ $(X \setminus A)$.Similarly we can prove (ii) ■.

Theorem 3.7: If A and B are any subsets of X then the following properties holds:

(i) If $A \subseteq B$ then Zc- $Int(A) \subseteq Zc$- $Int(B)$ and Zc-$cl(A) \subseteq Zc$-$cl(B)$
(ii) Zc- $Int(A) \cup Zc$ -$Int(B) \subseteq Zc$- $Int(A \cup B)$
(iii) Zc- $Int(A) \cap Zc$- $Int(B) \subseteq Zc$- $Int(A \cap B)$
(iv) Zc-$cl(A) \cup Zc$-$cl(B) \subseteq Zc$-$cl(A \cup B)$
(v) $Zccl(A \cap B) \subseteq Zc$-$cl(A) \cap Zc$-$cl(B)$ ■

Definition 3.8: Let A be a subset of a topological space (X,τ).Then a point $p \in X$ is called a Zc -limit point of a set $A \subseteq X$ if every Zc-open set $G \subseteq X$ containing p contains a point of A other than p. The set of all Zc-limit points of A is called a Zc-derived set of A and is denoted by Zc-$d(A)$.

Proposition 3.9: A subset A of a space X is Zc-closed iff it contains the set of all limit points. **Proof:** Assume that A is Zc-closed and if possible that x is a Zc-limit point of A which belongs to $X \setminus A$,then $X \setminus A$ is Zc-open set containing the Zc-limit point of A ,therefore $A \cap X \setminus A \neq \emptyset$ which is a contradiction. Conversely, assume that A contains the set of its Zc-limit points. For each $x \in X \setminus A$,there exists a Zc-open set U containing x such that $A \cap U = \emptyset$ then $x \in U \subset X \setminus A$.Thus $X \setminus A$ is Zc-open and hence A is Zc-closed ■.

Propositon 3.10: Let $A \subseteq (X,\tau)$.A point $x \in X$ is said to be in $Zc\text{-}cl(A)$ iff for each Zc-open set U containing x such that $U \cap A \neq \emptyset$.

Proof: Let $x \in Zc\text{-}cl(A)$ and suppose $U \cap A = \emptyset$,for some Zc-open set U, $x \in U$.Clearly $X \setminus U$ is Zc-closed and $A \subseteq (X \setminus U)$,thus $Zc\text{-}cl(A) \subseteq (X \setminus U)$ which then implies that $x \in (X \setminus U)$,a contradiction. Therefore $U \cap A \neq \emptyset$.Conversely, Suppose $U \cap A = \emptyset$ where U is Zc-open with $x \in U$ then $A \subseteq (X \setminus U)$ and $(X \setminus U)$ is Zc-closed with $x \notin (X \setminus U)$.Therefore $x \notin Zc\text{-}cl(A)$, a contradiction ■.

Definition 3.11: Let (X,τ) be a topological space and $A \subseteq X$. Then the Zc-boundary of A is defined by $Zc\text{-}b(A) = Zc\text{-}cl(A) \cap Zc\text{-}cl((X \setminus A)$ and is denoted by $Zc\text{-}b(A)$.

Theorem 3.12: Let $A \subseteq (X,\tau)$ then

(i) A is a Zc-clopen set iff $Zc\text{-}b(A) = \emptyset$

(ii) A is a Zc-closed set iff $Zc\text{-}b(A) \subseteq A$

(iii) A is a Zc-open set iff $A \cap Zc\text{-}b(A) = \emptyset$ ■

Lemma 3.13: Let A be a subset of (X,τ) then the following holds:

(i) $Zc\text{-}b(A) = Zc\text{-}b(X \setminus A)$
(ii) $Zc\text{-}b(A) = Zc\text{-}cl(A) \setminus Zc\text{-}Int(A)$
(iii) $Zc\text{-}b(A) \cap Zc\text{-}Int(A) = \emptyset$
(iv) $Zc\text{-}b(A) \cup Zc\text{-}Int(A) = Zc\text{-}cl(A)$ ■.

Definition 3.14: Let $A \subseteq (X,\tau)$.Then the set $X \setminus Zc\text{-}cl(A)$ is called the Zc-exterior of A and is denoted by $Zc\text{-}ext(A)$. A point $p \in X$ is called a Zc-exterior point of A, if it is a Zc-interior point of $X \setminus A$.

Lemma 3.15: If A and B are two subsets of (X,τ),then

(i) $Zc\text{-}ext(\emptyset) = X$ and $Zc\text{-}ext(X) = \emptyset$
(ii) $Zc\text{-}ext(A) = Zc\text{-}int\,(X \setminus A)$
(iii) $Zc\text{-}ext(A) \cap Zc\text{-}b(A) = \emptyset$
(iv) $Zc\text{-}ext(A) \cup Zc\text{-}b(A) = Zc\text{-}cl\,(X \setminus A)$
(v) If $A \subset B$ then $Zc\text{-}ext(B) \subset Zc\text{-}ext(A)$
(vi) $Zc\text{-}ext(A \cup B) \subset Zc\text{-}ext(B) \cup Zc\text{-}ext(A)$
(vii) $Zc\text{-}ext(A \cap B) \supset Zc\text{-}ext(A) \cap Zc\text{-}ext(B)$

4. *Zc* - Separation Axioms:

Definition 4.1: A topological space (X,τ) is said to be

(i) $Zc\text{-}T_0$ if for each pair of distinct points x, y in X, there exists a Zc-open set U such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.

(ii) (ii) $Zc\text{-}T_1$ if for each pair of distinct points x, y in X there exists two Zc-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$.(iii) $Zc\text{-}T_2$ if for each pair of distinct points x, y in X, there exists two disjoint Zc-open sets U and V containing x and y respectively.

Proposition 4.2: A topological space (X,τ) is $Zc\text{-}T_0$ iff for each pair of distinct points x, y of X , $Zc\text{-}cl(\{x\}) \neq Zc\text{-}cl(\{y\})$.

Proof: Let (X,τ) be a Zc-T_0 space and x, y be any two distinct points of X. Then there exists a Zc-open set U containing x or y, say x but not y. Then $X \setminus U$ is a Zc-closed set which does not contain x, but contains y. Since Zc-$cl(\{y\})$ is the smallest Zc-closed set containing y, Zc-$cl(\{y\}) \subseteq X \setminus U$ and hence $x \notin Zc$-$cl(\{y\})$.Consequently Zc-$cl(\{x\}) \neq Zc$-$cl(\{y\})$.Conversely, suppose $x, y \notin X$, $x \neq y$ and Zc- $cl(\{x\}) \neq Zc$-$cl(\{y\})$.Let p be a point of X such that $p \in Zc$-$cl(\{x\})$ but $p \notin Zc$-$cl(\{y\})$.We claim: $x \notin Zc$-$cl(\{y\})$.Suppose if $x \in Zc$-$cl(\{y\})$ then Zc-$cl(\{x\}) \subseteq Zc$-$cl(\{y\})$.This contradicts the fact that $p \notin Zc$-$cl(\{y\})$.Consequently x belongs to the Zc-open set $X \setminus Zc$-$cl(\{y\})$ to which y does not belong ■.

Proposition 4.3: A topological space (X,τ) is Zc-T_1 iff the singletons are Zc-closed.

Proof: Let (X,τ) be Zc-T_1 and x be any point of X. Suppose $y \in X \setminus \{x\}$,then $x \neq y$ and so there exists a Zc-open set U such that $y \in U$ but $x \notin U$. Consequently $y \in U \subseteq X \setminus \{x\}$ that is $X \setminus \{x\} = \bigcup\{U : y \in X \setminus \{x\}$ which is Zc-open. Conversely, Suppose $\{p\}$ is Zc-closed for every $p \in X$. Let $x, y \in X$ with $x \neq y$.Now $x \neq y$ implies $y \in X \setminus \{x\}$.Hence $X \setminus \{x\}$ is a Zc-open set containing y but not x. Similarly $X \setminus \{y\}$ is a Zc-open set containing x but not y. Hence X is a Zc-T_1 space ■.

Proposition 4.4: The following statements are equivalent

(i) X is Zc-T_2
(ii) Let $x \in X$. For each $y \neq x$, there exists a Zc-open set U containing x such that $y \notin Zc$-$cl(U)$
(iii) For each $x \in X$, $\bigcap\{Zc$-$cl(U) : U \in ZcO(X)$ and $x \in U\} = \{x\}$.

Proof: (i)⇒(ii) **:** Since X is Zc-T_2,by definition there exist disjoint Zc-open sets U and V containing x and y respectively. So $U \subseteq X \setminus V$. Therefore Zc-$cl(U) \subseteq X \setminus V$ .So $y \notin Zc$-$cl(U)$.

(ii) ⇒ (iii) **:** If for some $y \neq x$, we have $y \in Zc$-$cl(U)$ for every Zc-open set U containing x which then contradicts

(ii). (iii)⇒(i) **:** Let $x, y \in X$ and $x \neq y$. Then there exists Zc-open set U containing x such that $y \notin Zc$-$cl(U)$.Let $V = X \setminus Zc$-$cl(U)$ then $y \in V$ and $x \in U$ and also $U \cap V = \emptyset$ ■.

Definition 4.5: A space X is called

(i) θT_2-space iff for each $x \neq y$ in X there exists disjoint θ-open sets U, V such that $x \in U$ and $y \in V$.
(ii) Zc-*Regular* space iff for each x in X and a θ-closed set F such that $x \notin F$, there exists disjoint Zc-open sets open sets U, V such that $x \in U$ and $F \subseteq V$.
(iii) Zc*-*Regular* space iff for each $x \in X$ and a Zc-closed set F such that $x \notin F$, there exist disjoint sets U, V such that U is θ-open, V is Zc-open and $x \in U$, $F \subseteq V$.
(iv) *Almost* Zc*-*Regular* iff for each x in X and F a Zc-regular closed set such that $x \notin F$, there exists disjoint sets U, V such that U is θ-open, V is Zc-open and $x \in U$, $F \subseteq V$.
(v) Zc-*Normal* space iff for every disjoint θ-closed sets F_1, F_2 there exists disjoint Zc-open sets U,V such that $F_1 \subseteq U$, $F_2 \subseteq V$.

Proposition 4.6: A space X is Zc-*regular* iff for every $x \in X$ and each θ-open set U in X such that $x \in U$, there exists an Zc-open set V such that $V \subseteq Zc$-$cl(V) \subseteq U$.
Proof: Let X be a Zc-regular space and $x \in X$. Let U be θ-open in X such that $x \in U$. Clearly $\overline{U}$ is a θ-closed set and $x \notin \overline{U}$.Then there exist disjoint Zc-open sets V,W such that $x \in V$, $\overline{U} \subseteq W$ .Hence $x \in V \subseteq Zc$-$cl(V) \subseteq Zc$-$cl(\overline{W}) \subseteq \overline{W} \subseteq U$. Conversely, let F be a θ-closed set such that $x \notin F$.Clearly $\overline{F}$ is an θ-open set and $x \in \overline{F}$.Then there exists Zc-open set V such that $x \in V \subseteq Zc$-$cl(W) \subseteq \overline{F}$.Then $x \in V$, $F \subseteq Zc$-$cl(\overline{V})$ and thus V and Zc-$cl(\overline{V})$ are disjoint Zc-open sets. Hence X is a Zc-regular space ■.

Proposition 4.7:

(i) A space X is Zc*-*regular* iff for every $x \in X$ and each Zc-open set U in X such that $x \in U$, there exists a θ-open set V such that $x \in V \subseteq Zc$-$cl(V) \subseteq U$.

(ii) A space X is *Almost Zc-regular* iff for every $x \in X$ and each θ-regular open set U in X such that $x \in U$, there exists an Zc-open set V such that $x \in V \subseteq Zc\text{-}cl(\bar{V}) \subseteq U$.

(iii) A space X is *Almost Zc*-regular* iff for every $x \in X$ and each Zc-regular open set U in X such that $x \in U$, there exists a θ-open set V such that $x \in V \subseteq Zc\text{-}cl(V) \subseteq U$.

Proof: Similar to Proposition 4.6 ■.

Proposition4.8: A space X is called *Zc-normal* iff for every θ-closed set $F \subseteq X$ and each θ-open set U in X such that $F \subseteq U$ there exists an Zc-open set V such that $F \subseteq V \subseteq Zc\text{-}cl(V) \subseteq U$ ■ .

Proposition 4.9: If X is both *Zc-normal* and θT_2-space, then X is *Zc-regular*.
Proof: Let $x \in X$ and U be an θ-open set such that $x \in U$.Then $\{x\}$ is θ-closed subset of X. Thus there exists a Zc-open set V such that $\{x\} \subseteq V \subseteq Zc\text{-}cl(V) \subseteq U$. By Proposition 4.8, $x \in V \subseteq Zc\text{-}cl(V) \subseteq U$ and hence by Proposition 4.6, X is a Zc-regular space ■.

5.Generalized *Zc*- closed sets:

Definition 5.1: A subset A of a topological space (X,τ) is a generalized Zc-closed (briefly $g.Zc$-closed) if $Zc\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ). A is said to be generalized Zc-open (briefly $g.Zc$-open) if its complement $X \setminus A$ is generalized Zc-closed in (X,τ). The family of all generalized Zc-closed(resp., generalized Zc-open) are denoted by $g.ZcC(X)$ (resp., $g.ZcO(X)$).

Example 5.2: Let $X=\{a ,b,c\}$with $\tau =\{\emptyset , X,\{a\},\{b\},\{a,b \}\}, \tau^c =\{X, \emptyset ,\{b,c\},\{a,c\},\{c\}\}$. The family of **:** Z-open sets $ZO(X) = \{ X, \emptyset ,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\}\}$ Zc-open sets $ZcO(X) =\{\emptyset , X, \{b,c\},\{a,c\}\}$ Zc-closed sets $ZcC(X) = \{\emptyset , X,\{a\},\{b\}\}$. The collection of $g.Zc$-closed sets $g.ZcC(X) = \{ \emptyset, X, \{a\},\{b\},\{c\},\{b,c\},\{c,a\}\}$ $g.Zc$-open sets $g.ZcO(X) = \{ \emptyset , X , \{b,c\},\{a,c\},\{a,b\},\{a\},\{b\}\}$.

Theorem 5.3: (i) The arbitrary intersection of any $g.Zc$-closed subsets of X is $g.Zc$-closed set of X .
(ii) The arbitrary intersection of any $g.Zc$-open subsets of X need not be $g.Zc$-open of X.

Proof: (i) Let $\{A_i: i\in I \}$be any collection of $g.Zc$-closed subsets of X such that $\bigcap_{i=1}^{\infty} A_i \subseteq H$ and H be Zc-open in X .Given A_i is a $g.Zc$-closed subset of X for each $i\in I$,we have $Zc\text{-}cl(A_i) \subseteq H$ for all $i\in I$. Hence $\bigcap_{i=1}^{\infty} Zc - cl(A_i) \subseteq H$ for all $i\in I$ and thus $Zc\text{-}cl(\bigcap_{i=1}^{\infty} A_i) \subseteq H$. Therefore $\bigcap_{i=1}^{\infty} A_i$ is a $g.Zc$-closed set of X ■.
(ii) In example 5.2 ,the subsets $\{b,c\},\{c,a\}$ of X are $g.Zc$-open but their intersection $\{c\}$ is not a $g.Zc$-open set of X .

Remark 5.4: (i) The union of two $g.Zc$-closed subsets of X need not be a $g.Zc$-closed set of X as shown in example 5.2 in which the two subsets $\{b\},\{a\}$ of X are $g.Zc$-closed subsets ,but their union $\{a,b\}$ is not $g.Zc$- closed ■.

Theorem 5.5: A subset A of a space (X,τ) is $g.Zc$-closed iff for each $A\subseteq H$ and H is Zc-open there exists a Zc-closed set F of X such that $A\subseteq F \subseteq H$.

Proof: Assume that A is a $g.Zc$-closed subset of X, $A \subseteq H$ and H is a Zc-open set in X. Hence $Zc\text{-}cl(A)\subseteq H$. Put $F= Zc\text{-}cl(A)$.Thus we get $A\subseteq F \subseteq H$. Conversely, assume $A \subseteq H$ and H is Zc-open set. Given, there exists a Zc-closed set F of X such that $A\subseteq F \subseteq H$. So $A\subseteq Zc\text{-}cl(A) \subseteq F$ and hence $Zc\text{-}cl(A) \subseteq H$. Therefore A is $g.Zc$-closed ■.

Theorem 5.6: Let A be a $g.Zc$-closed subset of (X,τ).Then $Zc\text{-}cl(A)- A$ does not contain any non-empty closed sets.
Proof: Let F be a closed subset of $Zc\text{-}cl(A) – A$. Since $X – F$ is open, $A\subseteq X – F$ and A is $g.Zc$-closed, it follows that $Zc\text{-}cl(A) \subseteq X – F$ and thus $F \subseteq X - Zc\text{-}cl(A)$.This implies that $F \subseteq (X - Zc\text{-}cl(A)) \cap (Zc\text{-}cl(A)- A) = \emptyset$ and hence $F = \emptyset$ ■.

Corollary 5.7: Let A be a $g.Zc$-closed set. Then A is Zc-closed iff $Zc\text{-}cl(A) – A$ is closed.

Proof: Let A be a $g.Zc$-closed set. If A is Zc-closed then by theorem 5.6, $Zc\text{-}cl(A)- A = \emptyset$ which is closed set. Conversely, let $Zc\text{-}cl(A)- A$ be a closed set. Then by theorem 5.6, $Zc\text{-}cl(A)- A$ does not contain any non-empty closed sets. Since $Zc\text{-}cl(A)- A$ is closed subset of itself, then $Zc\text{-}cl(A)- A = \emptyset$ which implies that $A = Zc\text{-}cl(A)$ and so A is Zc-closed ■.

Corollary 5.8: If A is open and a $g.Zc$-closed set of X, then A is $g.Zc$-closed in X.
Proof: Let U be any open set of X so that $A \subseteq U$. Since A is open and a $g.Zc$-closed sets of X, we have $Zc\text{-}cl(A) \subseteq A$. Then $Zc\text{-}cl(A) \subseteq A \subseteq U$ and so A is a $g.Zc$-closed set ■.

Theorem 5.9: For any element $p \in X$ of a space X, the set $X \setminus \{p\}$ is a $g.Zc$-closed or Zc-open.
Proof: Suppose that $X \setminus \{p\}$ is not an Zc-open set. Then X is the only Zc-open set containing $X \setminus \{p\}$.Then $Zc\text{-}cl(X \setminus \{p\}) \subseteq X$ and thus $X \setminus \{p\}$ is $g.Zc$-closed in X ■.

Corollary 5.10: For a topological space (X,τ), every singleton of X is either $g.Zc$-open or Zc-closed.
Proof: Let (X,τ) be a topological space and $p \in X$. To prove that $\{p\}$ is either $g.Zc$-open or Zc-closed, enough to show $X \setminus \{p\}$ is either $g.Zc$-closed or Zc-open. Then the proof follows from theorem 5.9 ■.

Propositon 5.11: If A is a $g.Zc$-closed set of X such that $A \subseteq B \subseteq Zc\text{-}cl(A)$ then B is $g.Zc$-closed in X.
Proof: Let U be an open set of X such that $B \subseteq U$.Then $A \subseteq U$. Since A is a $g.Zc$-closed set of X, we have $Zc\text{-}cl(A) \subseteq U$. Now $Zc\text{-}cl(B) \subseteq Zc\text{-}cl(Zc\text{-}cl(A)) = Zc\text{-}cl(A) \subseteq U$.Therefore B is $g.Zc$-closed in X ■.

Proposition 5.12: If A is both open and a $g.Zc$-closed subset of a topological space (X,τ) then A is Zc- closed.
Proof: Assume that A is both an open and a $g.Zc$-closed subsets of X, then $Zc\text{-}cl(A) \subseteq A$. Hence A is Zc-closed ■.

Theorem 5.13: If A is both open and a $g.Zc$-closed subsets of X and F is a δ-closed set of X, then $A \cap F$ is $g.Zc$-closed in X.
Proof: Let A be an open and a $g.Zc$-closed subsets of X and F be a δ-closed set in X. Then by proposition 5.12 , A is Zc-closed. So, $A \cap F$ is Zc-closed and therefore $A \cap F$ is a $g.Zc$-closed set of X ■.

Theorem 5.14: (i) If A is δ-closed and B is a $g.Zc$-closed subset of a space X, then $A \cup B$ is $g.Zc$-closed.
Proof: Suppose that $A \cup B$ is a subset of an Zc-open set say H, then $A \subseteq H$ and $B \subseteq H$. Given B is $g.Zc$-closed, then $Zc\text{-}cl(B) \subseteq H$ and hence $A \cup B \subseteq A \cup Zc\text{-}cl(B) \subseteq H$ .But $A \cup Zc\text{-}cl(B)$ is a Zc-closed set. Hence there exist a Zc-closed set, $A \cup Zc\text{-}cl(B)$ of X such that $A \cup B \subseteq A \cup Zc\text{-}cl(B) \subseteq H$. By theorem 3.5, $A \cup B$ is $g.Zc$-closed.
(ii) If A is an δ-open and B is a $g.Zc$-open subset of a space X, then $A \cap B$ is $g.Zc$-open.
Proof: Obvious ■.

6. *Zc*-generalized closed sets:

Definition 6.1: A subset A of a topological space (X,τ) is a Zc- generalized closed (briefly $Zc.g$-closed) if $Zc\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is Zc-open in (X,τ). A is said to be Zc- generalized open (briefly $Zc.g$-open) if its complement $X \setminus A$ is Zc- generalized closed in (X,τ). The family of all Zc- generalized closed(resp., Zc-generalized open) are denoted by $Zc.gC(X)$ (resp., $Zc.gO(X)$).
Example 6.2: Let $X=\{a,b,c,d\}$ with $\tau=\{\emptyset, X,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{c,a\},\{a,b,c\}\}$, $\tau^c=\{X,\emptyset,\{b,c,d\},\{a,c,d\},\{a,b,d\},\{c,d\},\{a,d\},\{b,d\},\{d\}\}$. The family of Zc-open sets $ZcO(X)=\{\emptyset, X, \{a,d\},\{b,d\},\{c,d\},\{a,c,d\},\{a,b,d\},\{b,c,d\}\}$ Zc-closed sets $ZcC(X)=\{\emptyset, X,\{a,c\},\{b,c\},\{a,b\},\{b\},\{c\},\{a\}\}$. The collection of all $Zc.g$-closed sets $Zc.gC(X)=\{\emptyset, X, \{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{c,a\},\{c,d\},\{a,b,c\}\}$ $Zc.g$-open sets $Zc.gO(X)=\{\emptyset, X, \{b,c,d\},\{a,c,d\},\{a,b,d\},\{c,d\},\{b,d\},\{a,d\},\{a,b\},\{d\}\}$.
Propositon 6.3: The intersection of a $Zc.g$-closed set and a Zc-closed set is always $Zc.g$-closed ■.
Proposition 6.4: If A is Zc-open and $Zc.g$-closed then A is Zc-closed.
Proof: Suppose that A is Zc-open and $Zc.g$-closed. Since A is Zc-open and $A \subseteq A$ we have $Zc\text{-}cl(A) \subseteq A$, also $A \subseteq Zc\text{-}cl(A)$.Thus $Zc\text{-}cl(A) = A$. Hence A is Zc-closed ■.
Proposition 6.5: Let A be $Zc.g$-closed set in (X,τ) and $A \subseteq B \subseteq Zc\text{-}cl(A)$, then B is a $Zc.g$-closed set in X.
Proof: Let A be a $Zc.g$-closed set such that $A \subseteq B \subseteq Zc\text{-}cl(A)$. Let U be a Zc-open set of X so that $B \subseteq U$. Since A is $Zc.g$-closed, we have $Zc\text{-}cl(A) \subseteq U$. Now $Zc\text{-}cl(A) \subseteq Zc\text{-}cl(B) \subseteq Zc\text{-}cl(Zc\text{-}cl(A)) = Zc\text{-}cl(A) \subseteq U$. Hence $Zc\text{-}cl(B) \subseteq U$ where U is Zc-open. Thus B is a $Zc.g$-closed in X ■.

Lemma 6.6: For a space (X,τ) the following are equivalent:

(i) Every subset of X is $Zc.g$-closed.
(ii) $ZcO(X,\tau) = ZcC(X,\tau)$
Proof: (i) ⇒(ii)**:** Let $U \in ZcO(X,\tau)$.Then by hypothesis, U is $Zc.g$-closed which implies that $Zc\text{-}cl(U) \subseteq U$, so $Zc\text{-}cl(U) = U$. Hence U will be in a Zc-closed set. Also let $V \in ZcC(X,\tau)$. Then $X - V \in ZcO(X,\tau)$.Hence by hypothesis, $X - V$ is $Zc.g$-closed and then $X - V \in ZcC(X,\tau)$.Thus $V \in ZcO(X,\tau)$,then we have $ZcO(X,\tau) =$

$ZcC(X,\tau)$. (ii) $\Rightarrow$(i) **:** Let $A\subseteq (X,\tau)$ such that $A\subseteq U$ where $U\in ZcO\ (X,\tau)$. Then $U\in ZcC\ (X,\tau)$ and hence $Zc\text{-}cl(U)\subseteq U$ which implies A is $Zc.g$-closed.

The following example illustrate the above lemma ■.

Example 6.7: Let $X=\{a,b,c\}$with $\tau=\{\emptyset, X,\{a\},\{b\},\{a,b\},\{b,c\}\}$, $\tau^{c}=\{X,\emptyset,\{b,c\},\{a,c\},\{c\},\{a\}\}$. The family of Zc-open sets $ZcO(X)=\{\emptyset, X, \{b,c\},\{a\}\}$ Zc-closed sets $ZcC(X)=\{\emptyset, X,\{a\},\{b,c\}\}$ $Zc.g$-closed sets $Zc.gC(X)=\{\emptyset, X, \{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{c,a\}\}$

Proposition 6.8: A set A of a space X is $Zc.g$-closed iff $Zc\text{-}cl(A)\setminus A$ does not contain any non-empty Zc-closed set.
Proof: proof is similar to Theorem 5.6 ■.

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