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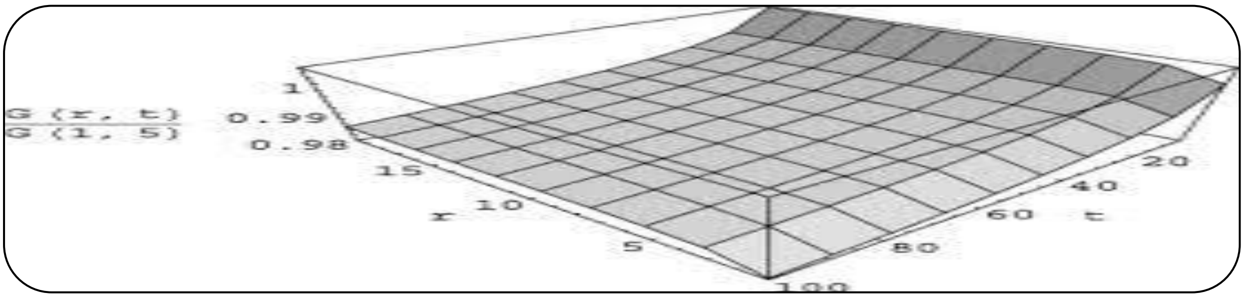

BEHAVIOUR OF INHOMOGENEOUS COSMOLOGICAL MODEL FILLED WITH NONTHERMALISED PERFECT FLUID

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ABSTRACT

The general solution of this system has not been found, but many interesting particular cases may be completely integrated providing solutions with all types of behaviors. some explicit exact solutions are given here. A general class of inhomogeneous cosmological model filled with nonthermalised perfect fluid by taking that the background spacetime admits two spacelike killing vectors and has separable metric coefficients. The singularity behaviour of these models depends on the choice of the metric functions and parameters. In the limit of vanishing heat flux, a number of previously known perfect fluid solutions follow as particular cases of this general class of solution. Here it is also considered the physical and geometrical features of these models. Apart from this exact solutions are also given.

KEY WORDS: heat flux generalization, singularities, spacetime, nonthermalised, metric coefficients

I. INTRODUCTION

The basic equations and formulas of the metric and proved that the metrics are generated by the solutions to a system of coupled first order ordinary differential equation. The general solution of this system has not been found, but many interesting particular cases may be completely integrated providing solutions with all types of behaviors. some explicit exact solutions are given here. Among these solutions there appeared singularity free family of solutions. It is proved that this family of solutions is the only one without singularities and, by means of a qualitative analysis of the differential equations, the general behavior of the general metric are obtained here. It is observed that these equations admit solutions in terms of elementary functions in some special cases.

KEY WORDS: heat flux generalization, singularities, spacetime, nonthermalised, metric coefficients

II. Explicit Solutions:

We have presented different classes of solutions, all of them for particular values of the parameters m, n, d, λ and e.

Case (1) For $\in = 0$ be the simplest family of solutions which implies

1. $T(t) = At + B$

2. $P = (N + C^{s5})^{s1}$,

3. $G = P^{s_2} C^{s_3}$,

4. $H = M C' P^{s_4}$,

where M, N are arbitrary constants, C be the arbitrary function of x, and

5. $s_1 = (1+2d)\ \{2(1+4d)s_2 + 4\lambda\}^{-1}$

6. $s_2 = (4d+1)^{-1}(-2\lambda \pm \sqrt{4\lambda^2 + 4d + 1})$

7. $s_3 = -(2d+1)/2d$

8. $s_4 = (1+2d)-1\ \{-4\lambda - (1+6d)s_2\}$

9. $s_5 = (1+4d)/4d$

By assuming d ≠ 0, 1+2d ≠ 0, 1+4d ≠ 0, one may evaluate the physical variables p, ρ, q as

10. $$16\pi\, p F^2 T^{2m} = A^2 \frac{(4m+1-n^2)}{2(At+B)^2} - \frac{(2d+1)^2(4d+1)}{16d^2 M^{2^{s_4}} C^2} - (C^{s_{5P^{-1/s_1}}} - 2)$$

11. $$16\pi\, \rho F^2 T^{2m} = A^2 \frac{(4m+1-n^2)}{2(At+B)^2} - \frac{(2d+1)(4d+1)(2d-3)}{16d^2 M^2 P^{2^{s_4}} C^2}(C^{s_{5p}\,^{-1/s_1}} - 2)$$

12. $$8\pi\, qF^2 T^{2m} = -\frac{A(1+2d)}{2MP^{\,s_4}\, C(At+B)d}\left[\frac{\{n-2\lambda+s_2(1-2m-2d)(1+4d)\}}{2\{2(1+4d)s_2+4\lambda\}}\right.$$

$$\left. -(1-2m-2d)\right]$$

In view of eqs. (10) – (12) with 1- 2m ≠2d or n ≠ 2λ, these solutions are a heat flux generalisation of Case 1 solution of (*Ruiz and Senovilla(1992*). For m = 2, n=3, N= -1, C(x) = $(1+x^2)^{3/5}$, d = - λ ≠ - 3/2 the above solution is recognised as heat flux counterpart of (*Davidson 1991*). Now we shall discuss some special cases:

Subcase d = 0, implies that

$$P = (N+C)^{s_1}$$

13. $$G = P^{s_2} C^{-1}$$

$$H = MC' P^{s_4} C^{-1}$$

where M, N are arbitrary constants and

$$s_1 = \frac{s_2}{(1+s_2^2)}$$

14. $$s_2^2 = \frac{1}{2}\left\{-1-4\lambda + \sqrt{(1+4)^2+4}\right\}$$

$$s_4 = -4\lambda - s_2$$

with λ < ¼. In the absence of heat flux the solution reduces to the case m = ½ discussed by Ruiz and Senovilla.

Subcase d = - ½ It implies

$$P = A_1 G^{s}$$

15. $$G = \left(2\lambda \pm \sqrt{4\lambda^2 - 1}\right) K_1 C + K_2$$

$$H = C'$$

where A_1, k_1, k_2 are constants and $\lambda^2 \geq$ ¼ and

16. $$s = \frac{1}{(2\lambda \pm \sqrt{4\lambda^2 - 1})}$$

Subcase d = - 1/4 It implies

$P = (ax + b)^{1/8\lambda}$

17. $$G = e^{-ax} P^{1/4\lambda}$$

$$H = e^{-ax} P^{\frac{1-32\lambda^2}{4\lambda}},$$

where a and b are arbitrary constants. In the absence of heat flux this solution reduces to m = ¾ discussed by Ruiz and Senovilla.

Subcase $4\lambda^2 + 4d + 1 = 0.$ It implies

$$P = \exp\ (ax)^{8\lambda^2/(1+4\lambda^2)}$$

18. $$G = P^{1/2\lambda}\ (ax)^{(1-4\lambda^2)/(1+4\lambda^2)},$$

$$H = P^{1/2\lambda}$$

In the absence of the heat flux this reduces to $4m = n^2 + 3$ discussed by Ruiz and Senovilla.

Case 2 For $\in \neq 0$, we have two cases (1) $\in = 1$ and $\in = -1$. For $\in = 1$, we obtain

$$P = e^{-s_1 \mu ax} f^{s_1}$$

19. $$G = e^{\mu ax} f$$

$$H = 1$$

Where

$$f = A_1 e^{\max} + A_2 e^{-\max}$$

$$s_1 = \sqrt{-2d}$$

20. $$m^2 = \frac{n + s_1 \mu^2}{s_1}$$

$$\lambda^2 = -\frac{(1+6d)}{32d}$$

$$\mu^2 = \frac{1}{4(1+2d)} \left\{ 2(2m-1) - \frac{n}{\sqrt{-2d}}(3+10d) \right\}$$

where -1/6 < d < 0 and A_1, A_2 are arbitrary constants. The physical parameters p, ρ, q are

21. $$32\pi\, p\, F^2\, T^{2m} = (4m+1-n^2)\left(\frac{\dot{T}^2}{T^2} - a^2\right) + a^2(4m-n^2-3) + \Big[a^2\mu^2\left\{1+6d-\sqrt{1+6d}\right\} + \frac{f'^2}{f^2}\left\{1+6d+\sqrt{1+6d}\right\} + 2a\mu\frac{f'}{f}(1+2d)\Big]$$

22.

$$32\pi\,\rho\, F^2 T^{2m} = a^2\left[2m+2-n^2+\frac{\sqrt{1+6d}}{2\sqrt{-2d}}n - \frac{(2d-3)}{(2d+1)}\left\{1-2m+2n\frac{\sqrt{1+6d}}{2\sqrt{-2d}}\right\}\right] + (4m+1-n^2)\left(\frac{\dot{T}^2}{T^2}-a^2\right) + \frac{(2d-3)}{(2d+1)}\Big[a^2\mu^2\left\{1+6d-\sqrt{1+6d}\right\} + \frac{f'^2}{f^2}\left\{1+6d-\sqrt{1+6d}\right\} + 2a\mu\frac{f'}{f}(1+2d)\Big]$$

23. $$16\pi\, qF^2T^{2m} = -\frac{\dot{T}}{T}\left[\frac{f'}{f}\left\{1-2m-2d+\sqrt{-2d}\left(n-\frac{\sqrt{1+6d}}{2\sqrt{-2d}}\right)\right\} + \mu a\left\{1-2m-2d-\sqrt{-2d}\left(n-\frac{\sqrt{1+6d}}{2\sqrt{-2d}}\right)\right\}\right]$$

These solutions are the heat flux generalisation of perfect fluid Case-2 solution discussed by Ruiz and Senovilla.

Case 3 : The case is given by relations

$\in = \pm 1$,

$1+4d+4\lambda^2 > 0$,

24. d = - ½ ,

d ≠ ¼ ,

$$2\lambda \pm \sqrt{1+4d+4\lambda^2} = \frac{n(1+2d)(1-4d)}{2m+2n\lambda-1+8n\lambda\, d}$$

The physical parameters p, ρ, q read in view of the following solution

$$P = f^{s_1}$$

25. $$G = f\, f'^{s_2}$$

H = 1,

Where

$$s_1 = 2\lambda \pm \sqrt{1 + 4d + 4\lambda^2}$$

26. $$s_2 = \frac{1 + 2d}{2d}$$

$$\frac{f''}{f} = -\frac{2 \in na^2 d}{2\lambda \pm \sqrt{1 + 4d + 4\lambda^2}}$$

27. $$32\pi\, pF^2 T^{2m} = \left[\left(4m + 1 - n^2\right)\left(\frac{\dot{T}^2}{T^2} - \in a^2\right)\right]$$

$$+ \in a^2\left[4m - n^2 - 3 - 4nds_2(s_1 - 4\lambda)\right] + 4n^2 a^4 d^2 \frac{s_2^4}{s_1^2}\left(\frac{f}{f'}\right)^2$$

28. $$32\pi\, F^2 T^{2m} = \in a^2\left[2m + 2 - n^2 + 2\lambda n - \frac{(2d-3)}{(2d+1)}\{1 - 2m + 2n\lambda + 4nds_2(s_1 - 4\lambda)\}\right]$$

$$+ (4m + 1 - n^2)\left(\frac{\dot{T}^2}{T^2} - \in a^2\right) + \frac{(2d-3)}{(2d+1)}\frac{s_2^2}{s_1^2} 4n^2 a^2 d^2 \left(\frac{f}{f'}\right)^2,$$

29. $$16\pi\, qF^2 T^{2m} = -\frac{\dot{T}}{T}\left(\frac{f}{f'}\right)\left[(1 - 2m - 2d + s_1 n - 2\lambda s_1)\frac{f'^2}{f^2} - \frac{2s_2}{s_1}(1 - 2m - 2d) \in na^2 d\right]$$

where the function f will take the form of trigonometric or hyperbolic function depending on the value of $\in$. These solutions are heat flux generalisation of perfect fluid case-3 solution discussed by Ruiz and Senovilla.

<u>Case 4</u> It is characterised by the relation

30. 2m = 1+ n, d + λ = 0

the metric functions are given as

$$P^2 = -2 \in na^2 dM^2 C^2 + NC^{(4d+1)/2d} - Y > 0$$

31. $$G = PC - \frac{1+2d}{2d}$$

$$H = \frac{MC'}{P}$$

The physical parameters are listed as

32. $$32\pi \rho F^2 T^{2m} = \in a^2 \left[4(1-m^2) - \frac{(2m-1)}{2d}(2d-3)(2d+1) \right]$$

$$+4m(2-m)(\frac{\dot{T}^2}{T^2} - \in a^2) - \frac{(2d+1)(4d+1)(2d-3)}{4d^2} Y$$

33. $$32\pi pF^2 T^{2m} = -\in a^2 \left[4(1-m^2) + \frac{n}{2d}(2d+1)^2 \right]$$

$$+4m(2-m)(\frac{\dot{T}^2}{T^2} - \in a^2) - \frac{(2d+1)^2(4d+1)}{4d^2} Y$$

34. $$32\pi qF^2 T^{2m} = (2d+1)(1-2m-2d)\frac{P\dot{T}}{MTCd}.$$

These solutions are heat flow generalisation of Case-4 perfect fluid solution of Ruiz and Senovilla. The spacetime of this class of solutions has well defined cylindrical symmetry and it has as large subclass of singularity free cosmological models with heat flow.

Case 5 In this case we have two particular solutions characterised by two subcases

Subcase (i) $\lambda + d = 0$

In this case metric variables are

$$P = f^{-4d} f'$$

$$G = f\ f^{'}$$

35. H = 1,

$$\frac{f^{''}}{f} = \frac{na^2}{8d} < 0$$

The physical variables are given as

36. $$32\pi\ pF^2T^{2m} = 4a^2(1-m^2) + 4m(2-m)(\frac{\dot{T}^2}{T^2} + a^2) + 4(1+2d)^2\frac{na^2}{8d}$$

$$32\pi\ \rho\ F^2T^{2m} = -4a^2(1-m^2) + 4m(2-m)(\frac{\dot{T}^2}{T^2} + a^2) + 4(1+2d)(2d-3)\frac{na^2}{8d}$$

$$8\pi\ qF^2T^{2m} = -(1+2d)(1-2m-2d)\frac{\dot{T}}{T}\ \frac{f^{'}}{f}$$

<u>Subcase (ii)</u> 2m = n+1

In this case the metric variables are

$$P = f^{'}$$

37. $$G = f^{-(1+2d)/2d}\ f^{'}$$

H = 1

$$\frac{f^{''}}{f} = 2na^2d < 0$$

The physical variables corresponding to this case :

38. $$32\pi\ pF^2T^{2m} = 4a^2(1-m^2) + 4m(2-m)(\frac{\dot{T}^2}{T^2} + a^2)$$

$$+ (1+2d)^2\left[\frac{(1+4d)}{4d^2}\ \frac{f^{'2}}{f^2} - 2(2m-1)a^2\right]$$

39. $$32\pi\,\rho\,F^2T^{2m} = -4a^2(1-m^2) + 4m(2-m)\left(\frac{\dot{T}^2}{T^2} + a^2\right) + (2d-3)(2d+1)\left[\frac{(1+4d)}{4d^2}\frac{f'^2}{f^2} - 2(2m-1)a^2\right]$$

40. $$8\pi\,qF^2T^{2m} = \frac{(1+2d)}{4d}(1-2m-2d)\frac{\dot{T}}{T}\frac{f'}{f}$$

III. PHYSICAL AND GEOMETRICAL FEATURES OF THE SOLUTIONS:

Some solutions generalising the (*Ruiz and Senovilla 1992*) perfect fluid solutions with the presence of heat flux. These solutions are very rich in singularity structure similar to the class of solutions given by Ruiz and Senovilla. There are solutions with big bang singularity only, solutions with both spacelike timelike singularities and singularity free solutions. Hence, it is observed that the presence of heat flux does not provide to any qualitative change in the singularity structure of the perfect fluid solutions investigated by Ruiz and Senovilla. From the solution presented here, it is obvious that the presence of heat flux has effects directly on the geometry of spacetime and physical variables of its matter content. However, there are number of solutions representing the equations of state p=kρ. Now let us see the singularity free models with heat flux as given by Case-4 defined by

41. T(t) = Cos h (at)

42. C(x) = Cos h (kar)

43. $\in$ = 1,

44. n $\geq$ 3,

where k be the real constant. The metric for these models may be expressed by replacing coordinates x and y by r and ϕ.

45. $$ds^2 = Cosh^{2m}(at)[Cosh\,(kar)]^{-(1+2d)}\left\{dt^2 - \frac{M^2k^2a^2}{p^2}\sinh^2(kar)\,dr^2\right\}$$

$$-[Cosh(at)]^{2-2m}[Cosh(kar)]^{-(1+2d)/2d}\,dz^2$$

$$-[Cosh(at)]^{2m}[Cosh(kar)]^{-(1+2d)/2d}\,p^2d\phi^2$$

Where

46. $p^2 = 2(1-2m)a^2 d\, M^2\, Cosh^2\,(kar) + N\left[Cosh(kar)\right]^{(4d+1)/2d} - Y$

with the range of coordinates

$$-\infty < T,$$

$$z < \infty$$

47. 0 ≤ r < ∞

0 ≤ ϕ ≤ π

The physical parameters for this class of solutions are obtained

48. $32\pi\rho\,[Cosh(at)]^{2m}[Cosh(kar)]^{-(1+2d)}$

$$= a^2\left\{4(1-m^2) - \frac{(2m-1)}{2d}(2d-3)(2d+1)\right\} + 4m(m-2)a^2 Sech^2(at)$$

$$-\frac{(2d+1)(4d+1)(2d-3)}{2d^2}Y,$$

49. $32\pi\, p\,[Cosh(at)]^{2m}[Cosh(kar)]^{-(1+2d)}$

$$= -a^2\left\{4(1-m^2) + \frac{n}{2d}(2d+1)^2\right\} + 4m(m-2)Sech^2(at)$$

$$-\frac{(2d+1)^2(4d+1)}{4d^2}Y$$

50. $32\pi\, q\,[Cosh(at)]^{2m}[Cosh(kar)]^{-(1+2d)} = \dfrac{(2d+1)(1-2m-2d)}{Md}aP.$

$$Tanh\,(at)\,Sech\,(kar)$$

This family gives a general solution including the cylindrically symmetric inhomogeneous singularity free perfect fluid with heat flux. Heat flux vanishes for

51. d = - ½

52. 1 – 2m = 2 d.

IV. CONCLUDING REMARKS:

We have obtained a general class of inhomogeneous cosmological model filled with nonthermalised perfect fluid by taking that the background space-time admits two spacelike killing vectors and has separable metric coefficients. The singularity behavior of these models depends on the choice of the metric functions and the parameters. In the limit of vanishing heat flux, a number of previously known perfect fluid solutions follow as particular cases of this general class of solution. We have also studied the physical and geometrical features of these models. Apart from this some new exact solutions are also given.

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